17. VEKTORI I KVADRATNE MATRICE

17.1 Opcenito o vektorima

Vektor je usmjerena duzina i zato ima: pocetak (hvatiste), kraj i smjer. Vektor se oznacava sa oznakom na pr.: $\vec{a}, \vec{r}, \overrightarrow{PQ}$

Duzina $\left|\overrightarrow{\mathrm{PQ}}\right|$ ili $|\vec{\mathrm{r}}|$ naziva se duzina vektora, intenzitet ili norma vektora $\overrightarrow{\mathrm{PQ}}$ ili $\vec{r}$.

Pravac na kojem lezi vektor je nosac vektora.

Kolinearni vektori su oni, koji leze na paralelnim pravcima. Za njih vrijedi $\vec{a} = k \cdot \vec{b}$ ili ako su suprotni vektori, tada vrijedi $\vec{a} = -k \cdot \vec{b}$. Vrijednost k je skalar.

Skalar je kategorija , broj, koji nema karakteristike vektora.

Nul-vektor je vektor sa duzinom 0 i kolinearan je sa svakim vektorom.

Jedinicni vektor (ort) je vektor sa intenzitetom 1. $\vec{a}_0 = \dfrac{\vec{a}}{|\vec{a}|} \Rightarrow |\vec{a}_0| = 1$

Dva vektora su jednaka ako imaju jednaku duzinu i smjer (orijentaciju).

Zbroj dva vektora je vektor: $\vec{a} + \vec{b} = \vec{c}$. Zbroj se dobije ulancavanjem dva vektora. Na kraj prvog translacijom se doda pocetak drugog vektora. Rezultat je vektor koji ima duzinu od pocetka prvog do kraja drugog vektora.

Oduzimanje vektora je dano izrazom: $\vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right) = \vec{c}$. Oduzimanje se izvodi tako sto se na pocetak prvog vektora translacijom doda pocetak drugog vektora. Rezultat, vektor $\vec{c}$, ima duzinu od kraja drugog do kraja prvog vektora (vidi zadatak u nastavku)

Kolinerani vektori su linearno zavisni.

Linearno nezavisni vektori su oni vektori za koje vrijedi: $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 = \vec{0}$

Linearno nezavisni vektori cine bazu vektorskog prostora: V^2 – za ravninu i V^3 – za prostor.

Baza vektorskog prostora dana je sa tri uredjena jedinicna vektora $\vec{i}, \vec{j}, \vec{k}$ koji su linearno nezavisni.

Svaki vektor se moze rastaviti na komponente. Za prostor V^3 ima oblik:

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \Rightarrow \vec{a} = \left(a_x, a_y, a_z\right)$$

Skalarni umnozak vektora je skalar: $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \cos\varphi$; φ – kut izmedju vektor $\vec{a}$ i $\vec{b}$.

Za $\varphi = 0$ ili $\vec{a} \cdot \vec{b} = \left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right) \cdot \left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right) = a_xb_x + a_yb_y + a_zb_z$

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1 \qquad \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$$

Vektorski umnozak dva vektora je vektor: $\vec{a} \times \vec{b} = \vec{c}$ koji je okomit na $\vec{a}$ i $\vec{b}$.

Vektori $\vec{a}, \vec{b}$ i $\vec{c}$ cine desni sustav. $|\vec{c}| = \left|\vec{a} \times \vec{b}\right| = |\vec{a}| \cdot \left|\vec{b}\right| \cdot \sin\left(\vec{a}, \vec{b}\right)$

Vektorski umnozak jedinicnih vektora u desnom sustavu:

$\vec{i}x\vec{i} = 0 \quad \vec{j}x\vec{j} = 0 \quad \vec{k}x\vec{k} = 0 \quad \vec{i}x\vec{j} = k \quad \vec{j}x\vec{k} = i \quad \vec{k}x\vec{i} = \vec{j}$

$\vec{j}x\vec{i} = -k \quad \vec{k}x\vec{j} = -\vec{i} \quad \vec{i}x\vec{k} = -\vec{j}$

$$\left|\vec{a}\times\vec{b}\right| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = S_{\square} \qquad \sin\varphi = \frac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|}$$

Apsolutna vrijednost vektorskog produkta dva vektora jednaka je povrsini paralelograma sto ga zatvaraju zadani vektori.

Mjesoviti umnozak vektora oznacava se sa $\left[\vec{a},\vec{b},\vec{c}\right] = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$

$$\left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\sin\varphi\cdot\left|\vec{c}\right|\cos\psi = V_{\square} \qquad \left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

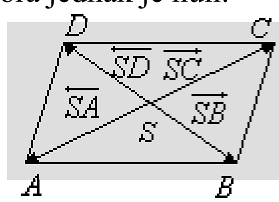
Apsolutna vrijednost mjesovitog umnoska vektora, jednaka je volumenu prizme koju zatvaraju vektori.

17.2 Osnovne operacije sa vektorima

1. Tocka S je sjeciste dijagonala paralelograma ABCD. Izracunaj vektorski zbroj $\overrightarrow{SA}+\overrightarrow{SB}+\overrightarrow{SC}+\overrightarrow{SD}.$

   Iz slike je vidljivo, da je $\overrightarrow{SA} = -\overrightarrow{SC}$ i $\overrightarrow{SB} = -SD \Rightarrow \overrightarrow{SA}+\overrightarrow{SB}+\overrightarrow{SC}+\overrightarrow{SD} = 0$

   Vektrorski zbroj zadanih vektora jednak je nuli.

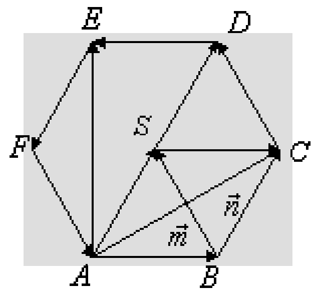


2. U pravilnom sesterokutu ABCDEF, poznati su vektori $\overrightarrow{AB} = \vec{m}$ i $\overrightarrow{BC} = \vec{n}.$

   Izrazi vektore $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$, $\overrightarrow{FA}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ i $\overrightarrow{AE}$ pomocu vektora $\vec{m}$ i $\vec{n}$.

   Iz slike je vidljivo, da je: $\overrightarrow{CD} = \overrightarrow{BS} = \overrightarrow{BA}+\overrightarrow{AS} = -\vec{m}+\vec{n} \Rightarrow \overrightarrow{DE} = -\vec{m} \quad \overrightarrow{EF} = -\vec{n}$

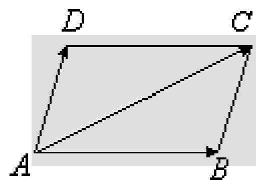
   $\overrightarrow{FA} = -\overrightarrow{CD} = \vec{m}-\vec{n} \quad \overrightarrow{AC} = \vec{m}+\vec{n} \quad \overrightarrow{AD} = 2\vec{n} \quad \overrightarrow{AE} = \overrightarrow{AD}+\overrightarrow{AD} = 2\vec{m}-\vec{n}$



3. Zadani su vektori prema slici. Izrcunaj zbroj vektora $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}$

Vidljivo je, da je $\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$ dijagonala pravokutnika ABCD

Prema tome je: $\underbrace{\overrightarrow{AB}+\overrightarrow{AD}}_{\overrightarrow{AC}}+\overrightarrow{AC}=\overrightarrow{AC}+\overrightarrow{AC}=2\overrightarrow{AC}$



4. Zadana su tri jedinicna vektora koji zadovoljavaju uvjet $\vec{a}+\vec{b}+\vec{c}=0$

Izracunaj zbroj $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

Iz zadanog uvjeta proizilazi, da zadani vektori cine jednakostranican trokut.

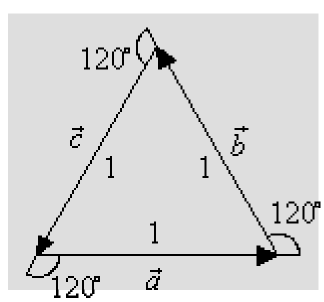
Kut izmedju vektora je u tom slicaju 120°. Dalje slijedi:

$$\vec{a}\cdot\vec{b}=|a||b|\cos 120^\circ=1\cdot 1\cdot\cos 120^\circ=\cos\left(90^\circ+30^\circ\right)=-\sin 30^\circ=-\frac{1}{2}$$

$$\vec{b}\cdot\vec{c}=|b||c|\cos 120^\circ=1\cdot 1\cdot\cos 120^\circ=\cos\left(90^\circ+30^\circ\right)=-\sin 30^\circ=-\frac{1}{2}$$

$$\vec{c}\cdot\vec{a}=|c||a|\cos 120^\circ=1\cdot 1\cdot\cos 120^\circ=\cos\left(90^\circ+30^\circ\right)=-\sin 30^\circ=-\frac{1}{2}$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=3\cdot\left(-\frac{1}{2}\right)=-\frac{3}{2}$$

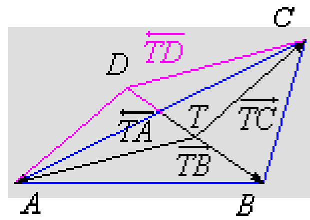


5. Zadan je trokut ABC i teziste u tocki T. Odredi zbroj vektora $\overrightarrow{TA}+\overrightarrow{TB}+\overrightarrow{TC}$

   Nadopunimo li trokut ATC u paralelogram ATCD mozemo postaviti:

   $$\overrightarrow{TA}+\overrightarrow{TB}+\overrightarrow{TC}=\underbrace{\overrightarrow{TA}+\overrightarrow{TC}}_{\overrightarrow{TD}}+\overrightarrow{TB}=\overrightarrow{TD}+\overrightarrow{TB}=0$$

   Vektori $\overrightarrow{TD}$ i $\overrightarrow{TB}$ su suprotni i njihov je zbroj nula.



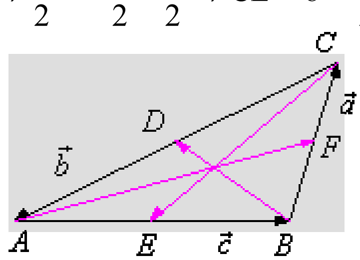
6. Zadana si tri vektora koji cine trokut: $\vec{a}=\overrightarrow{BC}$, $\vec{b}=\overrightarrow{CA}$ i $\vec{c}=\overrightarrow{AB}$. Izracunaj vektore tezisnica trokuta $\overrightarrow{AF}$, $\overrightarrow{BD}$ i $\overrightarrow{CE}$

   Iz $\triangle ABF$ imamo: $\overrightarrow{BF}=\frac{\vec{a}}{2}$ Iz $\triangle AFC$ imamo: $\overrightarrow{AF}=-\left(\vec{b}+\frac{\vec{a}}{2}\right)=\vec{c}+\frac{\vec{a}}{2}$

   Iz $\triangle BCD$ imamo: $\overrightarrow{BD}=\vec{a}+\frac{\vec{b}}{2}$

   $$\vec{b}=-\vec{c}-\vec{a}\Rightarrow\frac{\vec{b}}{2}=-\frac{\vec{c}}{2}-\frac{\vec{a}}{2}\quad\Rightarrow\overrightarrow{BD}=\vec{a}-\frac{\vec{c}}{2}-\frac{\vec{a}}{2}=\frac{\vec{a}-\vec{c}}{2}$$

   $$\overrightarrow{CE}=\vec{b}+\frac{\vec{c}}{2}\quad \vec{c}=-\vec{a}-\vec{b}\Rightarrow\frac{\vec{c}}{2}=-\frac{\vec{a}}{2}-\frac{\vec{b}}{2}\Rightarrow\overrightarrow{CE}=\vec{b}-\frac{\vec{a}}{2}-\frac{\vec{b}}{2}=\frac{\vec{b}-\vec{a}}{2}$$



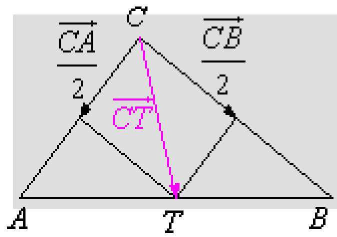
7. Iz vrha C trokuta povucena je tezisnica CT na bazu trokuta ABC u tocki T.

   Izrazi vektor tezisnice $\overrightarrow{CT}$ kao linearnu kombinaciju vektora stranica $\overrightarrow{CA}$ i $\overrightarrow{CB}$.

   Iz $\triangle ABC$ imamo: $(1)\ \overrightarrow{CT}=\overrightarrow{CA}+\overrightarrow{AT}\ (2)\ \overrightarrow{CT}=\overrightarrow{CB}+\overrightarrow{BT}$ zbrojimo (1)i(2)

   $2\cdot\overrightarrow{CT}=\overrightarrow{CA}+\overrightarrow{AT}+\overrightarrow{CB}+\overrightarrow{BT}$ iz slike vidimo: $\overrightarrow{AT}=-\overrightarrow{BT}$

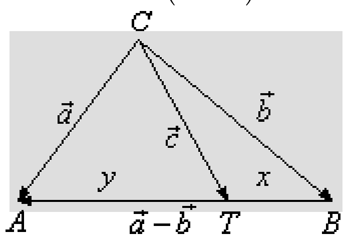
   $$2\cdot\overrightarrow{CT}=\overrightarrow{CA}+\overrightarrow{CB}\quad\Rightarrow\quad\overrightarrow{CT}=\frac{\overrightarrow{CA}+\overrightarrow{CB}}{2}=\frac{\overrightarrow{CA}}{2}+\frac{\overrightarrow{CB}}{2}$$



8. Iz tocke C povucena su tri vektora $\vec{a},\vec{b}$ i $\vec{c}$. Krajevi vektora leze na pravcu p, sa pripadajucim tockama A, B i T. Tocka T dijeli duzinu izmedju A i B u omjeru $x : y$, uz uvjet da je $x + y = 1$. Dokazi da je $\vec{c} = x\vec{a} + y\vec{b}$.

Iz zadatka 7. znamo, ako tocka T lezi na polovistu baze $\Rightarrow \vec{c} = \frac{1}{2}\left(\vec{a} + \vec{b}\right)$.

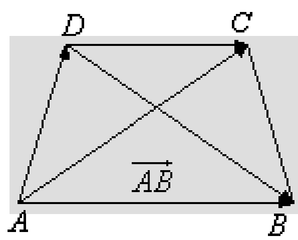
U ovom slucaju je: $\vec{c} = \overrightarrow{CB} + \overrightarrow{BT} = \vec{b} + x\left(\vec{a} - \vec{b}\right) = x\vec{a} + \left(1 - x\right)\vec{b} = x\vec{a} - y\vec{b}$



9. Zadan je trapez ABCD. Dokazi da su vektori dijagonala $\overrightarrow{AC}$ i $\overrightarrow{DB}$ kolinearni sa vektorom baze $\overrightarrow{AB}$.

Iz slike je vidljivo: $\overrightarrow{AC}=\overrightarrow{AD}+\overrightarrow{DC}=\overrightarrow{AD}+\lambda \cdot \overrightarrow{AB}$ $\qquad$ $\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AD}$

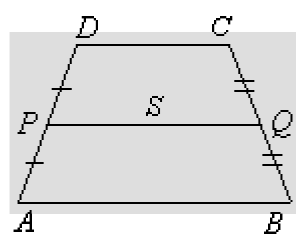
$$\overrightarrow{AC} + \overrightarrow{DB} = \overrightarrow{AD} + \lambda \cdot \overrightarrow{AB} + \overrightarrow{AB} - \overrightarrow{AD} = \left(\lambda + 1\right)\overrightarrow{AB}$$



10. Zadan je trapez ABCD. Dokazi da je sredisnjica trapeza jednaka polovici zbroja paralelnih stranica $\vec{S} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{DC}\right)$

Iz slike je vidljivo: $\overrightarrow{AP} + \overrightarrow{PQ} + \overrightarrow{QB} = \overrightarrow{AB}$ $\qquad$ $\overrightarrow{AP} = -\overrightarrow{DP}$

$\underline{\overrightarrow{DP} + \overrightarrow{PQ} + \overrightarrow{QC} = \overrightarrow{DC}}$ zbrojimo $2 \cdot \overrightarrow{PQ} = \overrightarrow{AB} + \overrightarrow{DC}$ $\qquad$ $\overrightarrow{PQ} = S = \frac{\overrightarrow{AB} + \overrightarrow{DC}}{2}$



11. Zadani su vektori $\vec{a}=3\vec{i}+4\vec{j}$ i $\vec{b}=2\vec{i}-\vec{j}$. Odredi intenzitet i smjer vektora $\vec{a},\vec{b},\left(\vec{a}+\vec{b}\right)$ i $\left(\vec{b}-\vec{a}\right)$.

Za $\vec{a}=3\vec{i}+4\vec{j}$ imamo: $|\mathrm{a}|=\sqrt{3^2+4^2}=\sqrt{25}=5 \Rightarrow \tan\varphi=\frac{4}{3} \Rightarrow \cos\varphi=\frac{3}{|a|}=\frac{3}{5}$

$\varphi=53.13^\circ$

Za $\vec{b}=2\vec{i}-\vec{j}$ imamo: $|\mathrm{b}|=\sqrt{2^2+1^2}=\sqrt{5} \Rightarrow \tan\varphi=-\frac{1}{2} \Rightarrow \cos\varphi=\frac{2}{|b|}=\frac{2}{\sqrt{5}}$

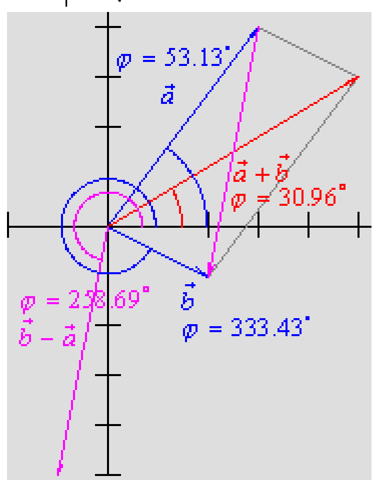
$\varphi=333.43^\circ$

Za $\vec{a}+\vec{b}$ imamo: $\left(3\vec{i}+4\vec{j}\right)+\left(2\vec{i}-\vec{j}\right)=5\vec{i}+3\vec{j}$; $|a+b|=\sqrt{5^2+3^2}=\sqrt{34}$

$\tan\varphi=\frac{3}{5} \Rightarrow \cos\varphi=\frac{5}{|a+b|}=\frac{5}{\sqrt{34}} \Rightarrow \varphi=30.96^\circ$

Za $\vec{b}-\vec{a}$ imamo: $\left(2\vec{i}-\vec{j}\right)-\left(3\vec{i}+4\vec{j}\right)=-\vec{i}-5\vec{j}$; $|b-a|=\sqrt{1^2+5^2}=\sqrt{26}$

$\tan\varphi=\frac{-5}{-1}=5 \Rightarrow \cos\varphi=\frac{5}{|b-a|}=\frac{-1}{\sqrt{26}} \Rightarrow \varphi=258.69^\circ$



12. Vektori $\vec{u}$ i $\vec{v}$ su kolinearni. Odredi broj x tako, da vektori $\vec{a}=(x-1)\vec{u}+\vec{v}$ i $\vec{b}=3\vec{u}+(x+1)\vec{v}$ budu kolinearni.

Iz uvjeta kolinearnosti dobijemo:

$$(x-1)\vec{u}+\vec{v}+3\vec{u}+(x+1)\vec{v}=0 \Rightarrow x(\vec{u}+\vec{v})+2\vec{v}+2\vec{u}=0 \Rightarrow x=-\frac{2(\vec{u}+\vec{v})}{(\vec{u}+\vec{v})}=-2$$

Uvrstimo u zadani izraz:

$\vec{a}=(-2-1)\vec{u}+\vec{v}=-3\vec{u}+\vec{v}$ $\quad\vec{b}=3\vec{u}+(-2+1)\vec{v}=3\vec{u}-\vec{v} \Rightarrow -\vec{a}$ $\quad\underline{\vec{a}=-\vec{b}}$

Za $x=2$ imamo:

$\vec{a}=(2-1)\vec{u}+\vec{v}=\vec{u}+\vec{v}$ $\quad\vec{b}=3\vec{u}+(2+1)\vec{v}=3(\vec{u}+\vec{v}) \Rightarrow 3\vec{a}$ $\quad\underline{\vec{b}=3\vec{a}}$

13. Vektor $\vec{a}$ rastavi na vektorske komponente okomite i paralelne vektoru $\vec{b}$.

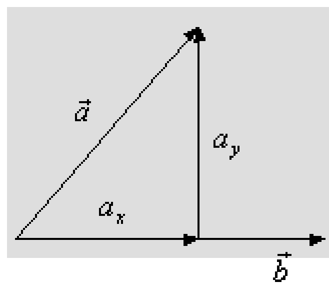
Rastasvimo vektor $\vec{a}$ na komponente a_x i a_y : $\vec{a} = \overrightarrow{a_x} + \overrightarrow{a_y}$ $\quad \overrightarrow{a_x} = m\vec{b}$

Iz uvjeta okomitosti: $\overrightarrow{a_y} \cdot \vec{b} = 0 \Rightarrow \overrightarrow{a_y} = \vec{a} - \overrightarrow{a_x} = \vec{a} - m\vec{b}$

$$\overrightarrow{a_y} \cdot \vec{b} = \left(\vec{a} - m\vec{b}\right) \cdot \vec{b} = \vec{a} \cdot \vec{b} - m\left|\vec{b}\right|^2 = 0 \Rightarrow m = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|^2} \text{ odnosno: } \overrightarrow{a_x} = m\vec{b} = \frac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|^2}\vec{b}$$

$\overrightarrow{a_y} = \vec{a} - m\vec{b} = \vec{a} - \dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|^2}\vec{b}$ $\qquad$ Skalar $\dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}$ je projekcija vektora $\vec{a}$ na vektor $\vec{b}$.

Vektor $\dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|}\left(\dfrac{\vec{b}}{\left|\vec{b}\right|}\right)$ je vektorska projekcija vektora $\vec{a}$ na vektor $\vec{b}$.



14. Vektori $\vec{a}$ i $\vec{b}$ su nekolinearni. Odredi realni broj x tako, da vektori $\vec{m} = \left(x+2\right)\vec{a} + \vec{b}$ i $\vec{n} = 2x\vec{a} + \left(x-1\right)\vec{b}$ postanu kolinearni suprotnog smjera.

Iz uvjeta kolinearnosti: $\vec{m} = k\ \vec{n}$ imamo: $x = -1$ i $x = 2$ odnosno;

Za $x = -1$:

$\vec{m} = \left(-1+2\right)\vec{a} + \vec{b} = \vec{a} + \vec{b}$ $\qquad \vec{n} = 2\left(-1\right)\vec{a} + \left(-1-1\right)\vec{b} = -2\vec{a} - 2\vec{b} \Rightarrow$ klinearni i suprotni

Za $x = 2$:

$\vec{m} = \left(2+2\right)\vec{a} + \vec{b} = 4\vec{a} + \vec{b}$ $\qquad \vec{n} = 2\left(2\right)\vec{a} + \left(2-1\right)\vec{b} = 4\vec{a} + \vec{b} \Rightarrow$ vektori su jednaki

15. Vektor $\vec{a} = 4\vec{i} + 3\vec{j}$ rastavi na vektore $\overrightarrow{a_1}$ paralelan i $\overrightarrow{a_2}$ okomit vektoru $\vec{b} = 3\vec{i} + \vec{j}$.

Iz ranijeg zadatka br.13 imamo $m = \dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{b}\right|^2} = \dfrac{4 \cdot 3 + 3 \cdot 1}{\left(3^2 + 1^2\right)} = \dfrac{15}{10} = \dfrac{3}{2}$

Sada je: $\overrightarrow{a_1} = m\vec{b} = \dfrac{3}{2}\left(3\vec{i} + \vec{j}\right) = \dfrac{9}{2}\vec{i} + \dfrac{3}{2}\vec{j}$

$$\overrightarrow{a_2} = \vec{a} - \overrightarrow{a_1} = \left(4\vec{i} + 3\vec{j}\right) - \left(\frac{9}{2}\vec{i} + \frac{3}{2}\vec{j}\right) = -\frac{1}{2}\vec{i} + \frac{3}{2}\vec{j}$$

16. Odredi vektor $\vec{k}$ tako, da za vektore $\vec{a}=2\vec{i}-\vec{j}$ i $\vec{b}=3\vec{i}+2\vec{j}$ vrijedi: $\vec{a}\cdot\vec{k}=7$ i $\vec{b}\cdot\vec{k}=7$.

Oznacimo $\vec{k}=m\vec{i}+n\vec{j}$ i upisimo skalarni umnozak dva vektora $\vec{a}\cdot\vec{k}=\left(2\vec{i}-\vec{j}\right)\left(m\vec{i}+n\vec{j}\right)$:

$$\vec{a}\cdot\vec{k}=\left(2\vec{i}-\vec{j}\right)\left(m\vec{i}+n\vec{j}\right)=2m\underbrace{\vec{i}\cdot\vec{i}}_{1}+2n\underbrace{\vec{i}\cdot\vec{j}}_{0}-m\underbrace{\vec{j}\cdot\vec{i}}_{0}-n\underbrace{\vec{j}\cdot\vec{j}}_{1}=\underline{2m-n=7}$$

Za drugi dio zadatka imamo:

$$\vec{b}\cdot\vec{k}=\left(3\vec{i}+2\vec{j}\right)\left(m\vec{i}+n\vec{j}\right)=3m\underbrace{\vec{i}\cdot\vec{i}}_{1}+3n\underbrace{\vec{i}\cdot\vec{j}}_{0}+2m\underbrace{\vec{j}\cdot\vec{i}}_{0}+2n\underbrace{\vec{j}\cdot\vec{j}}_{1}=\underline{3m+2n=7}$$

Rijesim te dvije jednadzbe sa dvije nepoznanice: $2m-n=7$

$$\underline{3m+2n=7}\Rightarrow m=3;\ n=-1$$

Trazeni vektor ima oblik: $\vec{k}=m\vec{i}+n\vec{j}=3\vec{i}-\vec{j}$.

17. Izracunaj umnozak $\left(3\vec{a}-2\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)$ ako je $|\vec{a}|=2$, $|\vec{b}|=3$ i $\sphericalangle\left(\vec{a},\vec{b}\right)=120^\circ$.

$$\left(3\vec{a}-2\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=3\vec{a}\vec{a}+3\vec{a}\vec{b}-2\vec{a}\vec{b}-2\vec{b}\vec{b}=3|\vec{a}|+3\vec{a}\vec{b}-|\vec{b}|=3\cdot2+3\vec{a}\vec{b}-2\cdot3$$

$$3\vec{a}\vec{b}=3|\vec{a}||\vec{b}|\cos\left(120^\circ\right)=3\cdot2\cdot3\cdot\left(-\frac{1}{2}\right)=-9\Rightarrow\left(3\vec{a}-2\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=-9$$

18. Odredi duzinu vektora $\vec{k}=3\vec{a}+2\vec{b}$ ako je $|\vec{a}|=2$, $|\vec{b}|=\sqrt{2}$ i $\sphericalangle\left(\vec{a},\vec{b}\right)=\frac{3\pi}{4}$.

Da bi odredili duzinu vektora $\vec{k}$, moramo izracunati njegovu apsolutnu vrijednst:

$$\left(\vec{k}\right)^2=\left(3\vec{a}+2\vec{b}\right)^2=\left(3\vec{a}\right)^2+2\left(3\vec{a}\right)\left(2\vec{b}\right)+\left(2\vec{b}\right)^2=9\vec{a}^2+12\vec{a}\vec{b}+4\vec{b}^2=$$

$$=9\cdot(2)^2+12\vec{a}\vec{b}+4\left(\sqrt{2}\right)^2 \qquad 12\vec{a}\vec{b}=12\cdot|\vec{a}||\vec{b}|\cos\left(\frac{3\pi}{4}\right)=12\cdot2\sqrt{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)=-24$$

$$\left(\vec{k}\right)^2=36-24+8=20\Rightarrow\ \vec{k}=\sqrt{20}=2\sqrt{5}$$

19. Odredi kut izmedju dijagonala paralelograma ABCD ako je $\overrightarrow{AB}=\vec{a}=4\vec{i}-3\vec{j}$ i $\overrightarrow{AD}=\vec{b}=6\vec{i}+\vec{j}$.

Iz slike je vidljivo da je:

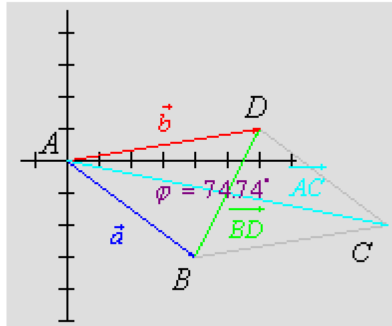
$$\overrightarrow{AC}=\vec{a}+\vec{b}=4\vec{i}-3\vec{j}+6\vec{i}+\vec{j}=10\vec{i}-2\vec{j} \qquad \left|\overrightarrow{AC}\right|=\sqrt{10^2+2^2}=\sqrt{104}$$

$$\overrightarrow{BD}=\vec{b}-\vec{a}=\left(6\vec{i}+\vec{j}\right)-\left(4\vec{i}-3\vec{j}\right)=2\vec{i}+4\vec{j} \qquad \left|\overrightarrow{BD}\right|=\sqrt{2^2+4^2}=\sqrt{20}$$

Iz skalarnog umnoska dobijemo: $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi\Rightarrow\varphi=ar\cos\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

$$\overrightarrow{AC}\cdot\overrightarrow{BD}=AC_iBD_i+AC_jBD_j=10\cdot2+(-2)\cdot4=20-8=12$$

$$\varphi=ar\cos\frac{12}{\sqrt{104}\sqrt{20}}=ar\cos\frac{12}{\sqrt{16\cdot130}}=ar\cos\frac{3}{\sqrt{130}}=74.74^\circ$$



20. Vektori $7\vec{a}-5\vec{b}$ i $\vec{a}+3\vec{b}$, su medjusobno okomiti kao i $7\vec{a}-2\vec{b}$ i $\vec{a}-4\vec{b}$. Izracunaj kut izmedju vektora $\vec{a}$ i $\vec{b}$.

Iz uvjeta okomitosti: $(7\vec{a}-5\vec{b})(\vec{a}+3\vec{b})=7|\vec{a}|^2-5\vec{a}\vec{b}-21\vec{a}\vec{b}-15|\vec{b}|^2=0$ i

$(7\vec{a}-2\vec{b})(\vec{a}-4\vec{b})=7|\vec{a}|^2-2\vec{a}\vec{b}-28\vec{a}\vec{b}+8|\vec{b}|^2=0$ sredimo i zbrojimo

$7|\vec{a}|^2+16\vec{a}\vec{b}-15|\vec{b}|^2=0$

$7|\vec{a}|^2-30\vec{a}\vec{b}+8|\vec{b}|^2=0$ oduzmimo drugu od prve jednadzbe

$46\vec{a}\vec{b}-23|\vec{b}|^2=0\Rightarrow \vec{a}\vec{b}=\frac{1}{2}|\vec{b}|^2\Rightarrow \vec{a}\vec{b}=|\vec{a}||\vec{b}|\cos\varphi=\frac{1}{2}|\vec{b}|^2\Rightarrow\cos\varphi=\frac{1}{2}\Rightarrow\varphi=60^\circ$

21. Vektori $\vec{a}+k\vec{b}$ i $\vec{a}-\vec{b}$ medjusobno su okomiti. Izracunaj faktor k ako je kut izmedju vektora $\vec{a}$ i $\vec{b}$, $\varphi=120^\circ$ i $|\vec{b}|=2|\vec{a}|$.

Iz uvjeta okomitosti: $(\vec{a}+k\vec{b})(\vec{a}-\vec{b})=0\Rightarrow|\vec{a}|^2+(k-1)\vec{a}\vec{b}-k|\vec{b}|^2=0$

nakon zamjene $|\vec{b}|=2|\vec{a}|$ i $\vec{a}\vec{b}=|\vec{a}||\vec{b}|\cos120^\circ=2|\vec{a}||\vec{a}|(-0.5)=-|\vec{a}|^2$

$|\vec{a}|^2+(k-1)\vec{a}\vec{b}-4k|\vec{a}|^2=0\Rightarrow|\vec{a}|^2+(k-1)(-|\vec{a}|^2)-4k|\vec{a}|^2=|\vec{a}|^2(2\text{-}5\text{k})=0\Rightarrow k=\frac{2}{5}$

22. Izracunaj $|\vec{a}+\vec{b}|$ i $|\vec{a}-\vec{b}|$ ako je poznat kut izmedju vektora $\sphericalangle(\vec{a},\vec{b})=60^\circ$ i ako je $|\vec{a}|=5$, $|\vec{b}|=8$.

$|\vec{a}+\vec{b}|^2=|\vec{a}|^2+2\vec{a}\vec{b}+|\vec{b}|^2=|\vec{a}|^2+2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2=5^2+2\cdot5\cdot8\cdot\frac{1}{2}+8^2=109$

$|\vec{a}+\vec{b}|=\sqrt{109}$

$|\vec{a}-\vec{b}|^2=|\vec{a}|^2-2\vec{a}\vec{b}+|\vec{b}|^2=|\vec{a}|^2-2|\vec{a}||\vec{b}|\cos\varphi+|\vec{b}|^2=5^2-2\cdot5\cdot8\cdot\frac{1}{2}+8^2=69$

$|\vec{a}-\vec{b}|=\sqrt{69}$

23. Izracunaj $(4\vec{a}-b)(2\vec{a}+3\vec{b})$ ako je $|\vec{a}|=2$, $|\vec{b}|=3$ i kut izmedju $\vec{a}$ i $\vec{b}$ iznosi 120°.

$$(4\vec{a}-b)(2\vec{a}+3\vec{b})=8|\vec{a}|^2+12\vec{a}\vec{b}-2\vec{a}\vec{b}+3|\vec{b}|^2=8\cdot 2^2+10\vec{a}\vec{b}-3\cdot 3^2=$$

$$=32-10|\vec{a}||\vec{b}|\cos\varphi-27$$

$$(4\vec{a}-b)(2\vec{a}+3\vec{b})=5-10\cdot 2\cdot 3(-0.5)=5-30=-25$$

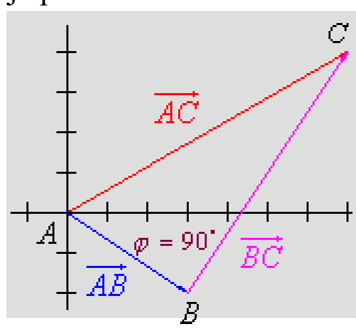
24. Zadani su vektori $\overrightarrow{AB}=3\vec{i}-2\vec{j}$ i $\overrightarrow{AC}=7\vec{i}+4\vec{j}$. Dokazi da trokut ABC koji cine vektori pravokutan.

$$\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}=7\vec{i}+4\vec{j}-(3\vec{i}-2\vec{j})=4\vec{i}+6\vec{j}$$

Iz uvjeta okomitosti: $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$

$$\overrightarrow{AB}\cdot\overrightarrow{BC}=(3\vec{i}-2\vec{j})(4\vec{i}+6\vec{j})=12\underbrace{\vec{i}\vec{i}}_{1}+18\underbrace{\vec{i}\vec{j}}_{0}-8\underbrace{\vec{i}\vec{j}}_{0}-12\underbrace{\vec{j}\vec{j}}_{1}=12-12=0$$

Vektori su okomiti i trokut je pravokutan.



25. Zadani su vektori $\vec{m}=\alpha\vec{a}+17\vec{b}$ i $\vec{n}=3\vec{a}-\vec{b}$ uz $|\vec{a}|=2$, $|\vec{b}|=5$ i $\sphericalangle(\vec{a},\vec{b})=\frac{2\pi}{3}$.

Izracunaj koeficijent α tako da vektori budu okomiti.

Iz uvjeta okomitosti: $\vec{m}\cdot\vec{n}=0\Rightarrow(\alpha\vec{a}+17\vec{b})\cdot(3\vec{a}-\vec{b})=0$

$$(\alpha\vec{a}+17\vec{b})\cdot(3\vec{a}-\vec{b})=3\alpha|\vec{a}|^2-\alpha\vec{a}\vec{b}+51\vec{a}\vec{b}-17|\vec{b}|^2=3\alpha 2^2+\vec{a}\vec{b}(51-\alpha)-17\cdot 5^2$$

$$\vec{a}\vec{b}=|\vec{a}||\vec{b}|\cos\left(\frac{2\pi}{3}\right)=2\cdot 5\cdot\left(-\frac{1}{2}\right)=-5$$

$$(\alpha\vec{a}+17\vec{b})\cdot(3\vec{a}-\vec{b})=12\alpha+(-5)(51-\alpha)-425=17\alpha-680=0\Rightarrow\alpha=\frac{680}{17}=40°$$

26. Zadani su vektori $\vec{a} = 2\vec{i} + (\alpha - 1)\vec{j} - \vec{k}$ i $\vec{b} = \alpha\vec{i} + \vec{j} + 2\vec{k}$. Izracunaj koeficijent α tako da vektori budu okomiti.

Iz uvjeta okomitosti:

$\vec{a} \cdot \vec{b} = 0 \Rightarrow a_i b_i + a_j b_j + a_k b_k = 0 \Rightarrow 2 \cdot \alpha + (\alpha - 1) \cdot 1 + (-1) \cdot 2 = 0$

$3\alpha = 3 \Rightarrow \alpha = 1$

27. Vektor $\vec{a} = \vec{i} + \alpha\vec{j} + \beta\vec{k}$ okomit je na vektore $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$ i $\vec{c} = -\vec{i} + \vec{j} + 2\vec{k}$. Izracunaj koeficijente α i β.

Iz uvjeta okomitosti:

$\vec{a} \cdot \vec{b} = 0 \Rightarrow a_i b_i + a_j b_j + a_k b_k = 1 \cdot 1 + \alpha(-2) + \beta = 0$

$\vec{a} \cdot \vec{c} = 0 \Rightarrow a_i b_i + a_j b_j + a_k b_k = \underline{1 \cdot (-1) + \alpha \cdot 1 + 2\beta = 0}$

Rijesimo te dvije jednadzbe: $1 - 2\alpha + \frac{1}{5} = 0 \Rightarrow \alpha = \frac{3}{5}$ $\qquad \beta = \frac{1}{5}$

28. Izracunaj vektor $\vec{c}$ koji je okomit na vektor $\vec{a} = 3\vec{i} + \vec{j} - 2\vec{k}$ i vektor $\vec{b} = 4\vec{i} - \vec{j} + 3\vec{k}$.

Iz vektorskog umnoska imamo: $\vec{c} = \vec{a} x \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -2 \\ 4 & -1 & 3 \end{vmatrix}$

Determinantu rijesimo skracenim postupkom:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -2 \\ 4 & -1 & 3 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 3 & 1 & -2 & 3 & 1 \\ 4 & -1 & 3 & 4 & -1 \end{vmatrix} = 3\vec{i}1 + 4(-2)\vec{j} + k3(-1) - 1\vec{k}4 - (-1)(-2)\vec{i} - 3\vec{j}3 =$$

$= \vec{i} - 17\vec{j} - 7\vec{k}$

Sada mozemo ispitati okomitost parova vektora:

$\vec{c} \cdot \vec{a} = c_i a_i + c_j a_j + c_k a_k = 1 \cdot 3 + (-17)1 + (-7)(-2) = 3 - 17 + 14 = 0$

$\vec{c} \cdot \vec{b} = c_i b_i + c_j b_j + c_k b_k = 1 \cdot 4 + (-17)(-1) + (-7)3 = 4 + 17 - 21 = 0$

29. Izracunaj povrsinu paralelograma odredjenog sa vektorima $\vec{a} = 3\vec{i} - 2\vec{j} + \vec{k}$ i $\vec{b} = -\vec{i} + \vec{j} - 2\vec{k}$.

Iz vektorskog umnoska imamo: $\vec{c} = \vec{a} x \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ -1 & 1 & -2 \end{vmatrix}$

Determinantu rijesimo skracenim postupkom:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ -1 & 1 & -2 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 3 & -2 & 1 & 3 & -2 \\ -1 & 1 & -2 & -1 & 1 \end{vmatrix} = \vec{i}(-2)(-2)1 + \vec{j}1(-1) + \vec{k}(-2)(-1) - \vec{i}1 \cdot 1 -$$

$$-\vec{j}3(-2) = 3\vec{i} + 5\vec{j} + \vec{k} \Rightarrow \left|\vec{a} x \vec{b}\right| = \sqrt{3^2 + 5^2 + 1^2} = \sqrt{35}$$

Povrsina paralelograma iznosi $\sqrt{35}$ kvadratnih jedinica

30. Izracunaj kut izmedju dijagonala parelelograma kojeg cine vektori $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ i $\vec{b} = \vec{i} - 3\vec{j} + \vec{k}$.

Iz slike vidimo: $\overrightarrow{AC} = \vec{d}_1 = \vec{a} + \vec{b} = 2\vec{i} + \vec{j} - \vec{k} + \vec{i} - 3\vec{j} + \vec{k} = 3\vec{i} - 2\vec{j}$

$$\overrightarrow{DB} = \vec{d}_2 = \vec{a} - \vec{b} = 2\vec{i} + \vec{j} - \vec{k} - \left(\vec{i} - 3\vec{j} + \vec{k}\right) = \vec{i} + 4\vec{j} - 2\vec{k}$$

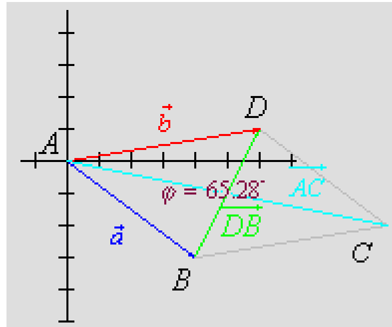
Kut medju dijagonalama iznosi: $\sin\varphi = \dfrac{\left|\vec{d}_1 x \vec{d}_2\right|}{\left|\vec{d}_1\right|\left|\vec{d}_2\right|}$

$$\left|\vec{d}_1 x \vec{d}_2\right| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 0 \\ 1 & 4 & -2 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 3 & -2 & 0 & 3 & -2 \\ 1 & 4 & -2 & 1 & 4 \end{vmatrix} = 4\vec{i} + 12\vec{k} + 2\vec{k} + 6\vec{j} = 4\vec{i} + 6\vec{j} + 14\vec{k}$$

$$\left|\vec{d}_1 x \vec{d}_2\right| = \sqrt{4^2 + 6^2 + 14^2} = \sqrt{248}$$

$$\left|\vec{d}_1\right| = \sqrt{3^2 + (-2)^2} = \sqrt{13} \qquad \left|\vec{d}_2\right| = \sqrt{1^2 + 4^2 + (-2)^2} = \sqrt{21}$$

$$\sin\varphi = \frac{\left|\vec{d}_1 x \vec{d}_2\right|}{\left|\vec{d}_1\right|\left|\vec{d}_2\right|} = \frac{\sqrt{248}}{\sqrt{13}\sqrt{21}} = \frac{\sqrt{248}}{\sqrt{273}} \Rightarrow \varphi = \arcsin\left(\frac{\sqrt{248}}{\sqrt{273}}\right) = 65.28^\circ$$



31. Izracunaj povrsinu parelelograma koji ima za dijagonale vektore $\vec{d}_1 = \vec{m} - \vec{n}$ i $\vec{d}_2 = 3\vec{m} - 4\vec{n}$ i ako su $\vec{m}$ i $\vec{n}$ jedinicni vektori pod kutem od $\varphi = 30^\circ$.

$$P = \frac{1}{2}\frac{\left|\vec{d}_1 x \vec{d}_2\right|}{\left|\vec{d}_1\right|\left|\vec{d}_2\right|} \Rightarrow$$

$$\vec{d}_1 x \vec{d}_2 = (\vec{m} - \vec{n})x(3\vec{m} - 4\vec{n}) = 3\underbrace{(\vec{m}x\vec{m})}_{0} - 4(\vec{m}x\vec{n}) + 3(\vec{n}x\vec{m}) + 4\underbrace{(\vec{n}x\vec{n})}_{0} =$$

$$= -4(\vec{m}x\vec{n}) + 3\underbrace{(\vec{n}x\vec{m})}_{-(\vec{m}x\vec{n})}$$

$$\left|\vec{d}_1 x \vec{d}_2\right| = -(\vec{m}x\vec{n}) = \vec{n}x\vec{m} = |\vec{n}||\vec{m}|\sin 30^\circ = 1\cdot 1\cdot\left(\frac{1}{2}\right) = \frac{1}{2} \Rightarrow P = \frac{1}{2}\frac{\left|\vec{d}_1 x \vec{d}_2\right|}{\left|\vec{d}_1\right|\left|\vec{d}_2\right|} = \frac{1}{2}\frac{\frac{1}{2}}{1\cdot 1} = \frac{1}{4}$$

32. Izracunaj povrsinu parelelograma koji ima stranice $\vec{a} = \vec{m} + 2\vec{n}$ i $\vec{b} = \vec{m} - 3\vec{n}$ i ako je $|\vec{m}| = 5$ i $|\vec{n}| = 3$ i kut medju njima $\alpha = 30^\circ$.

$$P = \left|\vec{a}x\vec{b}\right| = |\vec{a}||\vec{b}|\sin\varphi$$

$$\vec{a}x\vec{b} = (\vec{m} + 2\vec{n})x(\vec{m} - 3\vec{n}) = \underbrace{(\vec{m}x\vec{m})}_{0} - 3(\vec{m}x\vec{n}) + 2(\vec{n}x\vec{m}) - 6\underbrace{(\vec{n}x\vec{n})}_{0} = -3\underbrace{(\vec{m}x\vec{n})}_{-(\vec{n}x\vec{m})} + 2(\vec{n}x\vec{m})$$

$$\vec{a}x\vec{b} = 3(\vec{n}x\vec{m}) + 2(\vec{n}x\vec{m}) = 5(\vec{n}x\vec{m})$$

$$P = \left|\vec{a}x\vec{b}\right| = 5(\vec{n}x\vec{m}) = 5|\vec{n}||\vec{m}|\sin 30^\circ = 5\cdot 3\cdot 5\cdot\frac{1}{2} = \frac{75}{2}$$

33. Izracunaj $\vec{a}x\vec{b}$ ako je poznato: $\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}$ i $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$

$$\vec{c} = \vec{a}x\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} =$$

$$\vec{c} = \vec{i}\left[(-1)(-1) - (3)(2)\right] - \vec{j}\left[3\cdot(-1) - 2\cdot 2\right] + \vec{k}\left[3\cdot 3 - (-1)2\right] = \underline{-5\vec{i} + 7\vec{j} + 11\vec{k}}$$

34. Izracunaj povrsinu trokuta sa vrhovima u $A(2,3,5)$, B(4,2,-1), C(3,6,4).

Povrsina trokuta jednaka je $P_\triangle = \frac{1}{2}\vec{a}x\vec{b} \Rightarrow$ Stranice trokuta racunamo prema:

$$\overline{AB} = (4-2)\vec{i} + (2-3)\vec{j} + (-1-5)\vec{k} = 2\vec{i} - \vec{j} - 6\vec{k}$$

$$\overrightarrow{AC} = (3-2)\vec{i} + (6-3)\vec{j} + (4-5)\vec{k} = \vec{i} + 3\vec{j} - \vec{k}$$

$$P_\triangle = \frac{1}{2}\vec{a}x\vec{b} = P_\triangle = \frac{1}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -6 \\ 1 & 3 & -1 \end{vmatrix} = \frac{1}{2}\left\{\vec{i}\begin{vmatrix} -1 & -6 \\ 3 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -6 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}\right\} =$$

$$= \frac{1}{2}\left|19\vec{i} - 4\vec{j} + 7\vec{k}\right| \Rightarrow \ P_\triangle = \frac{1}{2}\sqrt{19^2 + (-4)^2 + 7^2} = \frac{1}{2}\sqrt{426}$$

35. Izracunaj volumen paralelopipeda sa stranicama $\vec{a} = 3\vec{i} - \vec{j}$; $\vec{b} = \vec{j} + 2\vec{k}$; $\vec{c} = \vec{i} + 5\vec{j} + 4\vec{k}$.

Volumen prizme je jednak povrsini baze pomnozene sa visinom:

Oznacimo li visinu sa vektorom ā, tada je povrsina baze jednaka vektorskom produktu vektora $\vec{b}$ i $\vec{c}$.

Volumen iznosi: $V = \left|\vec{a} \cdot (\vec{b} \times \vec{c})\right| \Leftrightarrow \left|\vec{a} \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{vmatrix}\right| = \left\|\begin{matrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{matrix}\right\|$ Zamijenimo:

$$V = \left|\vec{a} \cdot (\vec{b} \times \vec{c})\right| = \left\|\begin{matrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{matrix}\right\| = \left\|\begin{matrix} 3 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 5 & 4 \end{matrix}\right\| = |-20| = 20$$

Pravilo vrijedi i kada vektori zamijene mjesta: $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{c} \cdot (\vec{a} \times \vec{b})$

36. Izracunaj jednadzbu plohe koja lezi na vrhovima radij-vektora. Vrhovi radij-vektora imaju koordinate: A(3,1,-2), B(-1,2,4), C(2,-1,-1).

Vektori koji spajaju vrhove zadanih radij-vektora cine trokut koji lezi na trazenoj ravnini.

Oznacimo vektor, stranicu $\vec{a} = \overrightarrow{R_1R_2} = r_2 - r_1$; $\vec{b} = \overrightarrow{R_1R_3} = r_3 - r_1$; $\vec{c} = \overrightarrow{R_1R} = r - r_1$

Posto vektori leze na ravnini, vrijedi: $\overrightarrow{R_1R} \cdot \overrightarrow{R_1R_2} \times \overrightarrow{R_1R_3} = 0$

$$(r - r_1)\cdot(r_2 - r_1)\times(r_3 - r_1) = 0 \Leftrightarrow \begin{vmatrix} (r-r_1)_i & (r-r_1)_j & (r-r_1)_k \\ (r_2-r_1)_i & (r_2-r_1)_j & (r_2-r_1)_k \\ (r_3-r_1)_i & (r_3-r_1)_j & (r_3-r_1)_k \end{vmatrix} \equiv$$

$$\equiv \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

Odredimo radij-vektore:

$\mathrm{r} = x\vec{i} + y\vec{j} + z\vec{k}$ $\qquad$ $\mathrm{r}_1 = 3\vec{i} + \vec{j} - 2\vec{k}$ $\qquad$ $\mathrm{r}_2 = -\vec{i} + 2\vec{j} + 4\vec{k}$ $\qquad$ $\mathrm{r}_3 = 2\vec{i} - \vec{j} + \vec{k}$

$(r - r_1) = (x-3)\vec{i} + (y-1)\vec{j} + (z+2)\vec{k}$

$(r_2 - r_1) = -4\vec{i} + \vec{j} + 6\vec{k}$

$(r_3 - r_1) = -\vec{i} - 2\vec{j} + 3\vec{k}$

$(r - r_1)\cdot(r_2 - r_1)\times(r_3 - r_1) =$

$= \left\{(x-3)\vec{i} + (y-1)\vec{j} + (z+2)\vec{k}\right\}\cdot\left\{-4\vec{i} + \vec{j} + 6\vec{k}\right\}\times\left\{-\vec{i} - 2\vec{j} + 3\vec{k}\right\} = 0$

$$(r - r_1)\cdot(r_2 - r_1)\times(r_3 - r_1) = \left\{(x-3)\vec{i} + (y-1)\vec{j} + (z+2)\vec{k}\right\}\cdot\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & 6 \\ -1 & -2 & 3 \end{vmatrix} = 0$$

$(r - r_1)\cdot(r_2 - r_1)\times(r_3 - r_1) = \left\{(x-3)\vec{i} + (y-1)\vec{j} + (z+2)\vec{k}\right\}\cdot\left\{15\vec{i} + 6\vec{j} + 9\vec{k}\right\} = 0$

$\left\{(x-3)\vec{i} + (y-1)\vec{j} + (z+2)\vec{k}\right\}\cdot\left\{-4\vec{i} + \vec{j} + 6\vec{k}\right\} \Rightarrow 15(x-3) + 6(y-1) + 9(z+2) = 0$

$\underline{5x + 2y + 3z - 11 = 0}$ je trazena jednadzba ravnine.

Drugi nacin rjesavanja je koristenje ranije spomenute jednadzbe u obliku determinante:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = \begin{vmatrix} x-3 & y-1 & z+2 \\ -1-3 & 2-1 & 4+2 \\ 2-3 & -1-1 & 1+2 \end{vmatrix} = 0$$

$15(x-3) + 6(y-1) + 9(z+2) = 0 \Rightarrow$ Jednadzba ravnine: $\underline{5x + 2y + 3z - 11 = 0}$

37. Vektori $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ i $\vec{b} = \vec{i} - 3\vec{j} + \vec{k}$ cine dijagnale pralelograma. Izracunaj kut medju dijagonalama.

Prva dijagonala: $d_1 = \vec{a} + \vec{b} = 2\vec{i} + \vec{j} - \vec{k} + \vec{i} - 3\vec{j} + \vec{k} = 3\vec{\mathrm{i}} - 2\vec{\mathrm{j}}$

Druga dijagonala: $d_2 = \vec{a} - \vec{b} = 2\vec{i} + \vec{j} - \vec{k} - \left(\vec{i} - 3\vec{j} + \vec{k}\right) = \vec{i} + 4\vec{j} - 2\vec{k}$

$$\left|\vec{\mathrm{d}}_1 \times \vec{d}_2\right| = ab\sin\varphi \Rightarrow \sin\varphi = \frac{\left|\vec{\mathrm{d}}_1 \times \vec{d}_2\right|}{\mathrm{d}_1\mathrm{d}_2} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 0 \\ 1 & 4 & -2 \end{vmatrix}}{\sqrt{3^2 + (-2)^2}\sqrt{1^2 + 4^2 + (-2)^2}}$$

$$\sin\varphi = \frac{\left|4\vec{\mathrm{i}} + 6\vec{\mathrm{j}} + 14\vec{\mathrm{k}}\right|}{\sqrt{13}\sqrt{21}} = \frac{\sqrt{4^2 + 6^2 + 14^2}}{\sqrt{273}} = \frac{\sqrt{248}}{\sqrt{273}} \Rightarrow \varphi = \tan^{-1}\left(\frac{\sqrt{248}}{\sqrt{273}}\right) = 43.62°$$

38. Zadani su vektori: $\vec{a} = \vec{i} + \vec{j}$; $\vec{b} = 2\vec{i} - 3\vec{j} + \vec{k}$ i $\vec{c} = 4\vec{j} - 3\vec{k}$. Izracunaj: $\left(\vec{\mathrm{a}} \times \vec{b}\right) \times \vec{c}$; $\vec{\mathrm{a}} \times \left(\vec{b} \times \vec{c}\right)$ i zakljuci jesu li ta dva produkta jednaka.

$$\left(\vec{\mathrm{a}} \times \vec{b}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 2 & -3 & 1 \end{vmatrix} = \vec{i}(1-0) - \vec{j}(1-0) + \vec{k}(-3-2) = \vec{i} - \vec{j} - 5\vec{k}$$

$$\left(\vec{\mathrm{a}} \times \vec{b}\right) \times \vec{c} = \left\{\left(\vec{i} - \vec{j} - 5\vec{k}\right) \times \left(4\vec{j} - 3\vec{k}\right)\right\} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -5 \\ 0 & 4 & -3 \end{vmatrix} =$$

$$= \vec{i}(3+20) - \vec{j}(-3-0) + \vec{k}(4-0) \Rightarrow \qquad \left(\vec{\mathrm{a}} \times \vec{b}\right) \times \vec{c} = \underline{23\vec{i} + 3\vec{j} + 4\vec{k}}$$

$$\left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix} = \vec{i}(9-4) - \vec{j}(-6-0) + \vec{k}(8-0) = 5\vec{i} + 6\vec{j} + 8\vec{k}$$

$$\vec{\mathrm{a}} \times \left(\vec{b} \times \vec{c}\right) = \left\{\left(\vec{i} + \vec{j}\right) \times \left(5\vec{i} + 6\vec{j} + 8\vec{k}\right)\right\} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 5 & 6 & 8 \end{vmatrix} = \vec{i}(8-0) - \vec{j}(8-0) + \vec{k}(6-5)$$

$$\vec{\mathrm{a}} \times \left(\vec{b} \times \vec{c}\right) = \underline{8\vec{i} - 8\vec{j} + \vec{k}}$$

Zakljucujemo da je : $\left(\vec{\mathrm{a}} \times \vec{b}\right) \times \vec{c} \neq \vec{\mathrm{a}} \times \left(\vec{b} \times \vec{c}\right)$

39. Izracunaj jedinicni vektor $\vec{p}_0$ koji lezi na ravnini sto ga cine vektori $\vec{b}$ i $\vec{c}$ a okomit je na vektor $\vec{a}$.

$\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$; $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$; $\vec{c} = \vec{i} + \vec{j} - 2\vec{k}$

Trazeni vektor: $\vec{p} = p_x\vec{i} + p_y\vec{j} + p_z\vec{k}$ lezi na ravnini koju cine $\vec{a}$ i $\vec{b}$. Znaci vrijedi:

$$\left(\vec{b}\times\vec{c}\right)\cdot\vec{p} = 0 \Leftrightarrow \begin{vmatrix} p_x & p_y & p_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} p_x & p_y & p_z \\ 1 & 2 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 0$$

$$\vec{p} = p_x(-4+1) - p_y(-2+1) + p_z(1-2) = -3p_x + p_y - p_z = 0$$

Iz uvjeta $\vec{p} \perp \vec{a}$ slijedi: $\vec{p}\cdot\vec{a} = 0 \Rightarrow a_x p_x + a_y p_y + a_z p_z = 2p_x - 1p_y + 1p_z = 0$

Rijesimo sistem jednadzbi : $\left\{\begin{matrix} -3p_x + p_y - p_z = 0 \\ 2p_x - 1p_y + 1p_z = 0 \end{matrix}\right\} \Rightarrow \qquad \underline{p_x = 0} \qquad \underline{p_y = p_z}$

Jedinicni vektor: $|p_0| = \sqrt{p^2{}_x + p^2{}_y + p^2{}_z} = \sqrt{0 + p^2{}_y + p^2{}_y} = \sqrt{2p^2{}_y} \Rightarrow \sqrt{2}p_y$

$$|p_0| = \sqrt{2} \Leftrightarrow \vec{p}_0 = \frac{\vec{j}+\vec{k}}{\sqrt{2}}$$

40. Za zadane vektore: $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$; $\vec{b} = 3\vec{i} + \vec{k}$; $\vec{c} = \vec{j} - \vec{k}$ izracunaj $\left(\vec{a}+\vec{b}\right)\left(\vec{b}+\vec{c}\right)\times\vec{c}$

Zadatak mozemo izracunati na dva nacina:

1. $\left(\vec{a}+\vec{b}\right)\left(\vec{b}+\vec{c}\right)\times\vec{c} = \left(\vec{a}+\vec{b}\right)\left(\vec{b}\times\vec{c} + \overbrace{\vec{c}\times\vec{c}}^{0}\right) = \left(\vec{a}+\vec{b}\right)\left(\vec{b}\times\vec{c}\right) = \vec{a}\left(\vec{b}\times\vec{c}\right) + \overbrace{\vec{b}\left(\vec{b}\times\vec{c}\right)}^{0} =$

$$\left(\vec{a}+\vec{b}\right)\left(\vec{b}+\vec{c}\right)\times\vec{c} = \vec{a}\left(\vec{b}\times\vec{c}\right) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}\begin{vmatrix} 1 & -2 \\ 3 & 0 \\ 0 & 1 \end{vmatrix} =$$

$$= 0 - 0 + 3 - 0 - 1 - 6 = \underline{-4}$$

2. Uvedimo vektor: $\vec{u} = \left(\vec{a}+\vec{b}\right) = \left(2\vec{i} - \vec{j} + \vec{k}\right) + \left(\vec{i} + 2\vec{j} - \vec{k}\right) = 4\vec{i} - 2\vec{j} + 2\vec{k}$

$$\vec{v} = \left(\vec{b}+\vec{c}\right) = \left(\vec{i} + 2\vec{j} - \vec{k}\right) + \left(\vec{i} + \vec{j} - 2\vec{k}\right) = 3\vec{i} + \vec{j}$$

Zamijenimo: $\left(\vec{a}+\vec{b}\right)\left(\vec{b}+\vec{c}\right)\times\vec{c} = \vec{u}\cdot\vec{v}\times\vec{c} = \vec{u}\cdot\left(\vec{v}\times\vec{c}\right) = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} 4 & -2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$$\vec{u}\cdot\left(\vec{v}\times\vec{c}\right) = \begin{vmatrix} 4 & -2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}\begin{vmatrix} 4 & -2 \\ 3 & 1 \\ 0 & 1 \end{vmatrix} = -4 + 0 + 6 - 0 - 0 - 6 = \underline{-4}$$

17.3 Opcenito o kvadratnim matricama

Matrica je uredjena tablica realnih brojeva. Sastoji se od elemenata a_{ij} gdje je sa i oznacen broj retka a sa j broj stupca. Kvadratna matrica drugog reda ima dva reda i dva stupca.

Matrica se oznacava na slijedeci nacin: $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

a_{11} - je clan u prvom redu i prvom stupcu a_{12} - je clan u prvom redu i drugom stupcu

a_{21} - je clan u drugom redu i prvom stupcu a_{22} - je clan u drugom redu i drugom stupcu

Matrica sa samo jednim stupcem naziva se vektor matrica: $V=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Dvije matrice su jednake onda i samo onda ako su im odgovarajuci clanovi jednaki:

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \Rightarrow A = B \Rightarrow a_{11} = b_{11}, a_{12} = b_{12}, a_{21} = b_{21}, a_{22} = b_{22}$$

Transponirana matrica A^T, matrice A, se dobije tako, da se u matricu A^T upisu u redove clanovi stupaca matrice A. $\left(A^T\right)_{ij} = \left(A\right)_{ji}$ Matrica A oblika *mxn* postaje A^T oblika *nxm*

Zbroj matrica: Dvije matrice se zbrajaju tako, da se zbroje odgovarajuci clanovi matrice.

Rezultat se upisuje u matricu istog reda: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$

Umnozak broja i matrice: Broj se mnozi matricom tako, da se svaki elemet matrice pomnozi sa tim brojem. Rezultat je matrica istog reda: $\lambda \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} \\ \lambda a_{21} & \lambda a_{22} \end{bmatrix}$

Nul-matrica je matrica koja ima sve clanove jednake nuli. $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Zbrajanje matrica istog reda, svodi se na poznate zakone zbrajanja uz uvjet, da se racunske operacije vrse sa istim clanovima matrice. Rezultat je uvijek matrica jednakog reda:

$$A + B = B + A \qquad A - B = A + (-B) \qquad (A+B) + C = A + (B + C)$$

$$\lambda(A + B) = \lambda A + \lambda B \qquad (\lambda + \mu)A = \lambda A + \mu A \qquad (\lambda\mu)A = \lambda(\mu A)$$

Mnozenje matrica:

Mnozenjem dviju matrica A i B dobije se opet matrica, C. Mnozenje se vrsi na slijedeci nacin: Prvi clan rezultantne matrice, c_{11} dobije se tako, da se clanovi prvog reda matrice A pomnoze skalarno sa prvim stupcem matrice B. Drugi clan c_{12} se dobije skalarnim mnozenjem prvog reda matrice A sa drugim stupcem matrice B, itd.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} c_{11} = (a_{11}b_{11} + a_{12}b_{21}) & c_{12} = (a_{12}b_{12} + a_{12}b_{22}) \\ c_{21} = (a_{21}b_{11} + a_{22}b_{21}) & c_{22} = (a_{21}b_{12} + a_{22}b_{22}) \end{bmatrix}$$

Mnozenje matrica nije komutativno: AB nije jednako BA $AB \neq BA$

Regularna ili invertibilna matrica je ona matrica za koju vrijedi: $AB = BA = I$

Ako nije invertibilna, matrica je singularna

Inverzna matrica se oznacava sa A^{-1} i zadovoljava uvjet: $A^{-1}A = A \cdot A^{-1} = I$

Inverzna matrica zadovoljava matricnu jednadzbu

$$AX = B \,/\, A^{-1} \Rightarrow A^{-1}(AX) = A^{-1}B \Rightarrow (A^{-1}A)X = A^{-1}B \Rightarrow X = A^{-1}B$$

Inverzna matrica za kvadratnu matricu drugog reda $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ racuna se ovako:

1. Izracunamo determinantu detA i ako je A regularna, tj. $\det A \neq 0$, postoji inverzna matrica A^{-1}
2. Inverzna matrica jednaka je izrazu: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Inverzna matrica za kvadratnu matricu treceg reda $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ racuna se ovako:

1. Izracunamo determinantu detA i ako je A regularna, tj. $\det A \neq 0$, postoji inverzna matrica A^{-1}
2. Izracunamo transponiranu maricu matrice A tako sto izracunamo pod-determinantu svakog clana matrice A: $\text{Pdet}_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$ $\text{Pdet}_{12} = -\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$ $\text{Pdet}_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$$\text{Pdet}_{21} = -\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} \text{Pdet}_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} \text{Pdet}_{23} = -\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\text{Pdet}_{31} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} \text{Pdet}_{32} = -\begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} \text{Pdet}_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Transponirana matrica ima oblik: $A^T = \begin{bmatrix} \text{Pdet}_{11} & \text{Pdet}_{12} & \text{Pdet}_{13} \\ \text{Pdet}_{21} & \text{Pdet}_{22} & \text{Pdet}_{23} \\ \text{Pdet}_{31} & \text{Pdet}_{32} & \text{Pdet}_{33} \end{bmatrix}^T$

3. Inverzna matrica jednaka je izrazu: $A^{-1} = \frac{1}{\det A} A^T$
4. A^T se dobije tako da se elementi matrice zamijene njesta preslikavanjem oko glavne dijagonalne osi.

17.4 Rjesavanje kvadratnih matrica

1. Napisi matricu 3. reda kojoj je opci clan dan izrazom $a_{ij} = i - j + 1$.

$$\left\{\begin{matrix} a_{11} = 1-1+1=1 & a_{12} = 1-2+1=0 & a_{13} = 1-3+1=-1 \\ a_{21} = 2-1+1=2 & a_{22} = 2-2+1=1 & a_{23} = 2-3+1=0 \\ a_{31} = 3-1+1=3 & a_{32} = 3-2+1=2 & a_{33} = 3-3+1=1 \end{matrix}\right\} \Rightarrow A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

2. Izracunaj $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+2=3 & 2+(-1)=1 \\ (-1)+0=-1 & 0+1=1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

3. Izracunaj $\left(-\frac{2}{3}\right)\begin{bmatrix} 1 & 3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} \left(-\frac{2}{3}\right)\cdot 1 & \left(-\frac{2}{3}\right)\cdot 3 \\ \left(-\frac{2}{3}\right)\cdot(-6) & \left(-\frac{2}{3}\right)\cdot 4 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -2 \\ 4 & -\frac{8}{3} \end{bmatrix}$

4. Izracunaj $A + 2B - C$ za A= $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -3 & 2 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 6 & 4 \\ 0 & -5 & 1 \\ -3 & 4 & 8 \end{bmatrix}$

$$A + 2B - C = \begin{bmatrix} 1+2\cdot 1-0 & 2+2\cdot 2-6 & 2+2\cdot 1-4 \\ 0+2\cdot 0-0 & 1+2\cdot(-1)-(-5) & 3+2\cdot(-1)-1 \\ 3+2\cdot(-3)-(-3) & 0+2\cdot 2-4 & 2+2\cdot 2-8 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

5. Izracunaj $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 1-0+\frac{3}{2}\cdot 4 & 0-1+\frac{3}{2}\cdot 6 \\ -1-0+\frac{3}{2}\cdot 0 & 0-(-1)+\frac{3}{2}\cdot 8 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -1 & 13 \end{bmatrix}$

6. Izracunaj $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5\cdot 1+6\cdot 3 & 5\cdot 2+6\cdot 4 \\ 7\cdot 1+8\cdot 3 & 7\cdot 2+8\cdot 4 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$

7. Izracunaj $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ -2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix}$

$$\begin{bmatrix} 1\cdot 3+2\cdot(-2)+0\cdot 2 & 1\cdot 1+2\cdot 1+0\cdot 5 & 1\cdot(-1)+2\cdot 4+0\cdot 3 \\ (-1)\cdot 3+1\cdot(-2)+2\cdot 2 & (-1)\cdot 1+1+2\cdot 2 & (-1)\cdot(-1)+1\cdot 4+2\cdot 3 \\ (-2)\cdot 3+3\cdot(-2)+1\cdot 2 & (-2)\cdot 1+3\cdot 1+1\cdot 5 & (-2)\cdot(-1)+3\cdot 4+1\cdot 3 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 7 \\ -1 & 10 & 11 \\ -10 & 6 & 17 \end{bmatrix}$$

8. Izracunaj $B = 5A^2 - 3I$ ako je A=$\begin{bmatrix} 5 & 6 & 3 \\ 2 & \frac{3}{2} & 1 \\ 3 & 1 & 2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ jedinicna matrica

$$A^2 = A \cdot A = \begin{bmatrix} 5 & 6 & 3 \\ 2 & \frac{3}{2} & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 6 & 3 \\ 2 & \frac{3}{2} & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5\cdot 5+6\cdot 2+3\cdot 3 & 5\cdot 6+6\cdot\frac{3}{2}+3\cdot 1 & 5\cdot 3+6\cdot 1+3\cdot 2 \\ 2\cdot 5+\frac{3}{2}\cdot 2+1\cdot 3 & 2\cdot 6+\frac{3}{2}\cdot\frac{3}{2}+1\cdot 1 & 2\cdot 3+\frac{3}{2}\cdot 1+1\cdot 2 \\ 3\cdot 5+1\cdot 2+2\cdot 3 & 3\cdot 6+1\cdot\frac{3}{2}+2\cdot 1 & 3\cdot 3+1\cdot 1+2\cdot 2 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 46 & 42 & 27 \\ 16 & 15.25 & 9.5 \\ 23 & 21.5 & 14 \end{bmatrix} \Rightarrow B = 5A^2 - 3I = 5\cdot\begin{bmatrix} 46 & 42 & 27 \\ 16 & 15.25 & 9.5 \\ 23 & 21.5 & 14 \end{bmatrix} - 3\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 5\cdot 46-3\cdot 1 & 5\cdot 42-3\cdot 0 & 5\cdot 27-3\cdot 0 \\ 5\cdot 16-3\cdot 0 & 5\cdot 15.25-3\cdot 1 & 5\cdot 9.5-3\cdot 0 \\ 5\cdot 23-3\cdot 0 & 5\cdot 21.5-3\cdot 0 & 5\cdot 14-3\cdot 1 \end{bmatrix} = \begin{bmatrix} 227 & 210 & 135 \\ 80 & 73.25 & 47.5 \\ 115 & 107.5 & 67 \end{bmatrix} = \begin{bmatrix} 227 & 210 & 135 \\ 80 & \frac{293}{4} & \frac{95}{2} \\ 115 & \frac{215}{2} & 67 \end{bmatrix}$$

9. Izracunaj transponiranu matricu A^T za $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

10. Izracunaj 2A+B^T ako je A=$\begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

$$B = \begin{bmatrix} a_{11} = 2 & a_{12} = 0 & a_{13} = 1 \\ a_{21} = -1 & a_{22} = 1 & a_{23} = 3 \end{bmatrix} \qquad B^T = \begin{bmatrix} a_{11} = 2 & a_{21} = -1 \\ a_{12} = 0 & a_{22} = 1 \\ a_{13} = 1 & a_{23} = 3 \end{bmatrix}$$

$$2A + B^T = 2\cdot\begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ -2 & 5 \\ 1 & 5 \end{bmatrix}$$

11. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Determinanta detA: $\det A = 2 \cdot 2 - 3 \cdot 1 = 4 - 3 = 1 \neq 0$. Inverzna matrica postoji.

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

12. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}$

Determinanta detA: $\det A = -3 \cdot 4 - 2 \cdot 1 = -12 - 2 = -14 \neq 0$.Inverzna matrica postoji.

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = -\frac{1}{14}\begin{bmatrix} 4 & -1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} & \frac{1}{14} \\ \frac{1}{7} & \frac{3}{14} \end{bmatrix}$

13. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

Determinanta detA: $\det A = 2 \cdot 4 - (-3) \cdot 1 = 8 + 3 = 11 \neq 0$. Inverzna matrica postoji.

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix}$

14. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} 2i & 1 \\ 0 & 1+i \end{bmatrix}$ $i =$ imaginatna jedinica, $\sqrt{-1}$

Determinanta detA: $\det A = 2i \cdot (1+i) - 0 = 2i + 2i \cdot i = 2i + 2i^2 = 2i(1+i) \neq 0$. A^{-1}postoji.

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{2i(1+i)}\begin{bmatrix} 1+i & -1 \\ 0 & 2i \end{bmatrix} = \begin{bmatrix} \frac{1+i}{2i(1+i)} & \frac{-1}{2i+2i^2} \\ 0 & \frac{2i}{2i(1+i)} \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} \frac{1}{2i} \cdot \frac{i}{i} & \frac{1}{2i-2} \cdot \frac{(i+1)}{(i+1)} \\ 0 & \frac{2i}{2i(i+1)} \end{bmatrix} = \begin{bmatrix} \frac{i}{-2} & \frac{i+1}{2(i-1)(i+1)} \\ 0 & \frac{1}{1+i} \end{bmatrix} = \begin{bmatrix} -\frac{i}{2} & \frac{1+i}{4} \\ 0 & \frac{1}{1+i} \end{bmatrix}$$

15. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$

Determinanta detA: $\det A = \sin x \cdot \sin x + \cos x \cos x = 1 \neq 0$. Inverzna matrica postoji.

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = 1\begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix} = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$

Inverzna matrica $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = 1\begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix} = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$

16. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -2 \\ 4 & 0 & 2 \end{bmatrix}$

Izracunajmo determinantu A: $\det A = \det\begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -2 \\ 4 & 0 & 2 \end{bmatrix} = (-)(-1)\begin{vmatrix} 1 & -2 \\ 4 & 2 \end{vmatrix} = 2 + 8 = 10 \neq 0$

Pod-determinante matrice izgledaju ovako:

$\text{Pdet}_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & 2 \end{bmatrix} = 0\cdot 2 - (-2)\cdot 0 = 0$

$\text{Pdet}_{12} = -\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} = -\begin{bmatrix} 1 & -2 \\ 4 & 2 \end{bmatrix} = -\{(1\cdot 2) - (-2)\cdot 4\} = -10$

$\text{Pdet}_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 0 \end{bmatrix} = 1\cdot 0 - 0\cdot 4 = 0$

$\text{Pdet}_{21} = -\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} = -\begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix} = -\{(-1)\cdot 2 = 3\cdot 0\} = -2$

$\text{Pdet}_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} = 2\cdot 2 - 3\cdot 4 = -8$

$\text{Pdet}_{23} = -\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} = -\begin{bmatrix} 2 & -1 \\ 4 & 0 \end{bmatrix} = -\{2\cdot 0 - (-1)\cdot 4\} = -4$

$\text{Pdet}_{31} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} = (-1)(-2) - 3\cdot 0 = 2$

$\text{Pdet}_{32} = -\begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} = -\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} = -\{2\cdot(-2) - 3\cdot 1\} = 7$

$\text{Pdet}_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = 2\cdot 0 - (-1)\cdot 1 = 1$

Matrica prije transponiranja ima oblik:

$$A = \begin{bmatrix} \text{Pdet}_{11} & \text{Pdet}_{12} & \text{Pdet}_{13} \\ \text{Pdet}_{21} & \text{Pdet}_{22} & \text{Pdet}_{23} \\ \text{Pdet}_{31} & \text{Pdet}_{32} & \text{Pdet}_{33} \end{bmatrix} = \begin{bmatrix} 0 & -10 & 0 \\ 2 & -8 & -4 \\ 2 & 7 & 1 \end{bmatrix}$$

Transponirajmo matricu A: $A^T = \begin{bmatrix} 0 & -10 & 0 \\ 2 & -8 & -4 \\ 2 & 7 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & 2 \\ -10 & -8 & 7 \\ 0 & -4 & 1 \end{bmatrix}$

Inverzna matrica $A^{-1} = \frac{1}{\det A} A^T = \frac{1}{10}\begin{bmatrix} 0 & 2 & 2 \\ -10 & -8 & 7 \\ 0 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{2}{10} & \frac{2}{10} \\ -1 & -\frac{8}{10} & \frac{7}{10} \\ 0 & -\frac{4}{10} & \frac{1}{10} \end{bmatrix}$

17. Izracunaj inverznu matricu, matrice $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 8 & 9 \end{bmatrix}$

Izracunajmo: $\det A = \begin{vmatrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 2 & 8 & 9 & 2 & 8 \end{vmatrix} = 1\cdot5\cdot9 + 2\cdot6\cdot2 + 3\cdot4\cdot8 - 3\cdot5\cdot2 - 1\cdot6\cdot8 - 2\cdot4\cdot9 =$

$\det A = 15 \neq 0.$

Pod-determinante matrice izgledaju ovako:

$\text{Pdet}_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix} = 45 - 48 = -3$

$\text{Pdet}_{12} = -\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} = -\begin{bmatrix} 4 & 6 \\ 2 & 9 \end{bmatrix} = -\{36 - 12\} = -24$

$\text{Pdet}_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 2 & 8 \end{bmatrix} = 32 - 10 = 22$

$\text{Pdet}_{21} = -\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} = -\begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix} = -\{18 - 24\} = 6$

$\text{Pdet}_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 9 \end{bmatrix} = 9 - 6 = 3$

$\text{Pdet}_{23} = -\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} = -\begin{bmatrix} 1 & 2 \\ 2 & 8 \end{bmatrix} = -\{8 - 4\} = -4$

$\text{Pdet}_{31} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} = 12 - 15 = -3$

$\text{Pdet}_{32} = -\begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} = -\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} = -\{6 - 12\} = 6$

$\text{Pdet}_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} = 5 - 8 = -3$

Matrica prije transponiranja ima oblik:

$$A=\begin{bmatrix} \text{Pdet}_{11} & \text{Pdet}_{12} & \text{Pdet}_{13} \\ \text{Pdet}_{21} & \text{Pdet}_{22} & \text{Pdet}_{23} \\ \text{Pdet}_{31} & \text{Pdet}_{32} & \text{Pdet}_{33} \end{bmatrix}=\begin{bmatrix} -3 & -24 & 22 \\ 6 & 3 & -4 \\ -3 & 6 & -3 \end{bmatrix}$$

Transponirajmo matricu A^T: $\begin{bmatrix} -3 & -24 & 22 \\ 6 & 3 & -4 \\ -3 & 6 & -3 \end{bmatrix}^T=\begin{bmatrix} -3 & 6 & -3 \\ -24 & 3 & 6 \\ 22 & -4 & -3 \end{bmatrix}$

$$A^{-1}=\frac{1}{\det A}A^T=\frac{1}{15}\begin{bmatrix} -3 & 6 & -3 \\ -24 & 3 & 6 \\ 22 & -4 & -3 \end{bmatrix}=\begin{bmatrix} -\frac{3}{15} & \frac{6}{15} & -\frac{3}{15} \\ -\frac{24}{15} & \frac{3}{15} & \frac{2}{15} \\ \frac{22}{10} & -\frac{4}{15} & \frac{-3}{15} \end{bmatrix}=\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} & -\frac{3}{5} \\ -\frac{8}{5} & \frac{1}{5} & \frac{2}{5} \\ \frac{22}{15} & -\frac{4}{15} & -\frac{1}{5} \end{bmatrix}$$

18. Rijesi matricnu jednadzbu $AX=B$ ako je $A=\begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ i $B=\begin{bmatrix} 8 & 1 \\ 2 & 3 \end{bmatrix}$

Rjesiti jednadzbu znaci: $AX=B \Rightarrow X=A^{-1}B$ Odredimo naj prije A^{-1}

$$\det A=\det\begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix}=14-12=2\neq 0 \quad A^{-1}=\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{2}\begin{bmatrix} 7 & -4 \\ -3 & 2 \end{bmatrix}=\begin{bmatrix} \frac{7}{2} & -2 \\ -\frac{3}{2} & 1 \end{bmatrix}$$

$$X=A^{-1}B=\begin{bmatrix} \frac{7}{2} & -2 \\ -\frac{3}{2} & 1 \end{bmatrix}\begin{bmatrix} 8 & 1 \\ 2 & 3 \end{bmatrix}=\begin{bmatrix} \frac{7}{2}\cdot 8+(-2)\cdot 2 & \frac{7}{2}\cdot 1+(-2)\cdot 3 \\ -\frac{3}{2}\cdot 8+1\cdot 2 & -\frac{3}{2}\cdot 1+1\cdot 3 \end{bmatrix}=\begin{bmatrix} 24 & -\frac{5}{2} \\ -10 & \frac{3}{2} \end{bmatrix}$$

19. Rijesi matricnu jednadzbu $AXB=C$ ako je $A=\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $B=\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ i $C=\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

Prvi korak rjesenja se svodi na: $AXB=C \quad \Rightarrow \quad XB=A^{-1}C$

$$\det A=\det\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}=4-3=1\neq 0 \qquad A^{-1}=\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{1}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

$$XB=A^{-1}C\Rightarrow X\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}=\begin{bmatrix} -4-3 & 8+1 \\ 6+6 & -12-2 \end{bmatrix}=\begin{bmatrix} -7 & 9 \\ 12 & -14 \end{bmatrix}$$

$$X\underbrace{\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}}_{D}=\underbrace{\begin{bmatrix} -7 & 9 \\ 12 & -14 \end{bmatrix}}_{C}\Rightarrow X=CD^{-1}$$

Inverzna matrica D^{-1}: $\det D = \begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = 9 - 10 = -1 \neq 0 \Rightarrow D^{-1} = \frac{1}{-1}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$$X = CD^{-1} = \begin{bmatrix} -7 & 9 \\ 12 & -14 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -7\cdot 3 + 9\cdot 5 & -7\cdot 2 + 9\cdot 3 \\ 12\cdot 3 - 14\cdot 5 & 12\cdot 2 - 14\cdot 3 \end{bmatrix} = \begin{bmatrix} 24 & 13 \\ -34 & -18 \end{bmatrix}$$

20. Pomnozi: AB= $\begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3\cdot 2 + 1\cdot 3 & 3\cdot 1 + 1\cdot 0 & 3\cdot 1 + 1\cdot 1 \\ 2\cdot 2 + 1\cdot 3 & 2\cdot 1 + 1\cdot 0 & 2\cdot 1 + 1\cdot 1 \\ 1\cdot 2 + 0\cdot 3 & 1\cdot 1 + 0\cdot 0 & 1\cdot 1 + 0\cdot 1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 4 \\ 7 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix}$