

11. POLINOMI I RACIONALNI IZRAZI

Pojednostavi zadane izraze:

- $3x + 2x - 5y = 5x - 5y = 5(x - y)$
- $6a + 5c + 2a - c = 8a + 4c$
- $2(a + 2x) = 2a + 4x$
- $[a^2b - ab + (ab - 2a^2b)] = a^2b - ab + ab - 2a^2b = -a^2b$
- $[a^2b - ab + (ab - 2a^2b)] - \{[(3a^2b + b) - (4ab - 2a^2b)] - b\} =$
 $= [a^2b - ab + ab - 2a^2b] - \{[3a^2b + b - 4ab + 2a^2b] - b\} =$
 $= -a^2b - \{5a^2b + b - 4ab - b\} = -a^2b - 5ab + 4ab = -6a^2b + 4ab$
- $-(4 - x) - [(5x - 7) - (6x + 2)] = -4 + x - (5x - 7 - 6x - 2) = 2x + 5$
- $8c - \{5 - [2 - (3 + 4c)]\} = 8c - \{5 - [2 - 3 - 4c]\} =$
 $= 8c - \{5 + 4 + 4c\} = 8c - 6 - 4c = 4c - 6$
- $5p - (q - 2p) - [3q - (p - q)] = 5p - q + 2p - [3q - p + q] = 7p - q - 4q + p = 8p - 5q$
- $-2\{-(4 - x^2) - [3 + (4 - x^2)]\} = -2\{-4 + x^2 - [3 + 4 - x^2]\} =$
 $= -2\{-4 + x^2 - 7 + x^2\} = -2\{-11 + 2x^2\} = 22 - 4x^2$
- $a^2 - 2(x - 5 - (7 - 2(a^2 - 2x) - 3x)) = a^2 - 2(x - 5 - (7 - 2a^2 + 4x - 3x)) =$
 $= a^2 + 24 - 2a^2 = -a^2 + 24$
- $\left[\left(b + \frac{4}{3}a\right) + 2\left(b - \frac{2}{3}a\right)\right] - \left[\left(b + \frac{4}{3}a\right) - \left(b - \frac{2}{3}a\right)\right] =$
 $= \left(b + \frac{4}{3}a + 2b - \frac{4}{3}a\right) - \left(b + \frac{4}{3}a - b + \frac{2}{3}a\right) = (3b) - \left(\frac{6}{3}a\right) = 3b - 2a$

Mate Vijuga: Rijeseni zadaci iz matematike za srednju skolu

$$12. \quad 3(2a^2x)^3(-ax) = 3(8a^6x^3)(-ax) = -3(8a^7x^4) = -24a^7x^4$$

$$13. \quad 2s^3(-st^4)^2(4s^2t) = 2s^3(s^2t^8)(4s^2t) = 2s^3(4s^4t^9) = 8s^7t^9$$

$$14. \quad (2+x)(3-x)(x-1) = (2+x)(3x-3-x^2+x) = (2+x)(-x^2+4x-3) = \\ = -x^3+2x^2+5x-6$$

$$15. \quad 3x(x+2)^2(2x-1) = 3x(x^2+4x+4)(2x-1) = (3x^3+12x^2+12x)(2x-1) = \\ = 6x^4-3x^3+24x^3-12x^2+24x^2-12x = 6x^4+21x^3+12x^2-12x$$

$$16. \quad \frac{16x^3y^3}{4xy} = \frac{16x^3y^3}{4xy} = 4x^2y^3$$

$$17. \quad \frac{4a^2+8a}{2a} = \frac{4a^2}{2a} + \frac{8a}{2a} = 2a+4$$

$$18. \quad \frac{4x^3y-8x^3y^2+2x^2y}{2x^2y} = \frac{4x^3y}{2x^2y} - \frac{8x^3y^2}{2x^2y} + \frac{2x^2y}{2x^2y} = 2x-4xy+1$$

$$19. \quad (6x^2+x-2) \div (2x-1) = 2x+4x+2$$

$$\begin{array}{r} \underline{(-4x^2 + 2x)} \\ 2x^2 + 3x - 2 \\ \underline{-2x^2 - x - 2} \\ 4x - 2 \\ \underline{-4x + 2} \end{array}$$

$$20. \quad (4x^3+6x^2+1) \div (2x-1) = 2x^2+4x+2 \quad \text{Ostatak: } 3$$

$$\begin{array}{r} \underline{(-4x^3 + 2x^2)} \\ 8x^2 + 1 \\ \underline{-8x^2 + 4x + 1} \\ 4x + 1 \\ \underline{-4x + 2} \\ 3 \end{array}$$

21. $(2x^2 + 7x + 3) \div (x + 3) = 2x + 1$

$$\begin{array}{r} \underline{-2x^2 + \quad 6 + 3} \\ x + 3 \\ \underline{-x + \quad 3} \end{array}$$

22. $(x^2 - 3x + 2) \div (x - 2) = x - 1$

$$\begin{array}{r} \underline{-x^2 - \quad 2x + 2} \\ -x + 2 \\ \underline{-\quad x + \quad 2} \end{array}$$

23. $(8x^3 - 14x^2 + x) \div (2x - 3) = 4x^2 - x - 1$ Ostatak: -3

$$\begin{array}{r} \underline{-8x^3 - \quad 12x^2 + x} \\ -2x^2 + x \\ \underline{-\quad 2x^2 + \quad 3x} \\ -2x \\ \underline{-\quad 2x + \quad 3} \\ -3 \end{array}$$

24. $(4x^2 + 23x + 15) \div (4x + 3) = x + 5$

$$\begin{array}{r} \underline{-4x^2 + \quad 3x + 15} \\ 20x + 15 \\ \underline{-20x + \quad 15} \end{array}$$

25. $(3x^3 + 19x^2 + 16x - 20) \div (3x - 2) = x^2 + 7x + 10$

$$\begin{array}{r} \underline{-3x^3 - \quad 2x^2} \\ 21x^2 + 16x - 20 \\ \underline{-21x^2 - \quad 14x} \\ 30x - 20 \\ \underline{-30x - \quad 20} \end{array}$$

26. $(x^3 + 8) \div (x + 2) = x^2 - 2x + 4$

$$\begin{array}{r} \underline{-x^3 + \quad 2x^2} \\ -2x^2 + 8 \\ \underline{-\quad 2x^2 - \quad 4x} \\ 4x + 8 \\ \underline{-\quad 4x + \quad 8} \end{array}$$

27. $(3a^2 - 5ab + 2b^2) \div (a - 3b) = 3a + 4x$ Ostatak: $14b^2$

$$\begin{array}{r} \underline{-3a^2 - \quad 9ab} \\ 4ab + 2b^2 \\ \underline{-\quad 4ab - \quad 12b^2} \\ 14b^2 \end{array}$$

28. $(8a^5 + 4a^3m^2e^2 - am^4e^4) \div 8a^4 = \frac{8a^5}{8a^4} + \frac{4a^3m^2e^2}{8a^4} - \frac{am^4e^4}{8a^4} = a + \frac{m^2e^2}{2a} = \frac{m^4e^4}{8a^3}$

29. Pitanje : Nadji dva broja koji imaju zbroj 6 a produkt 8

Odgovor: 4,2 $n^2 - 6n + 8 = (n - 4)(n - 2)$

30. Nadji dva broja koji imaju zbroj +5 a produkt

	+5	a produkt	+6	(3,2)
	-5	a produkt	-6	(-6,1)
	-5	a produkt	+4	(-2,-3)
	+2	a produkt	-8	(4,-2)
	+4	a produkt	+3	(3,1)
	-2	a produkt	+8	(-4,2)
	+3	a produkt	-10	(5,-2)
	-7	a produkt	+6	(-6,-1)

31. $\frac{5x+15}{x^2-a} = \frac{5(x+3)}{(x-3)(x+3)} = \frac{5}{x-3}$

32. $\frac{8n^2+24n+18}{2n^2+n-3} = \frac{2(4n^2+12+3^2)}{(2n-1)(2n+3)} = \frac{2(2n+3)^2}{(2n-1)(2n+3)} = \frac{2(2n+3)}{(2n-1)}$

$$33. \quad \frac{x-2}{x^2-3x+2} = \frac{x-2}{(x-1)(x-2)} = \frac{1}{x-1}$$

$$34. \quad \frac{25x^2-9y^2}{10x+6y} = \frac{(5x-3y)(5x+3y)}{2(5x+3y)} = \frac{5x-3y}{2}$$

$$35. \quad \frac{x^2+10x+25}{x^2+4x-5} = \frac{(x+5)(x+5)}{(x+5)(x-1)} = \frac{(x+5)}{(x-1)} \quad x \neq -5, 1$$

$$33. \quad \frac{x-2}{x^2-3x+2} = \frac{x-2}{(x-1)(x-2)} = \frac{1}{x-1}$$

$$34. \quad \frac{25x^2-9y^2}{10x+6y} = \frac{(5x-3y)(5x+3y)}{2(5x+3y)} = \frac{5x-3y}{2}$$

$$35. \quad \frac{x^2+10x+25}{x^2+4x-5} = \frac{(x+5)(x+5)}{(x+5)(x-1)} = \frac{(x+5)}{(x-1)} \quad x \neq -5, 1$$

$$36. \quad \frac{m^2-m-20}{3m-15} = \frac{(m-5)(m+4)}{3(m-5)} = \frac{(m+4)}{3} \quad m \neq -4$$

$$37. \quad \frac{n^2-7n+12}{n^2-n-12} = \frac{(n-3)(n-4)}{(n-4)(n+3)} = \frac{(n-3)}{(n+3)} \quad n \neq -3, 4$$

$$38. \quad \frac{9a^2-4}{9a^2-3a-2} = \frac{(3a-2)(3a+2)}{(3a-2)(3a+1)} = \frac{(3a+2)}{(3a+1)} \quad a \neq -\frac{1}{3}, \frac{2}{3}$$

$$39. \quad \frac{q^2+q-20}{q^2-11q+28} = \frac{(q+5)(q-4)}{(q-4)(q-7)} = \frac{(q+5)}{(q-7)} \quad q \neq 4, -5, 7$$

$$40. \quad \frac{6d^2-d-15}{12d^2-23d+5} = \frac{(2d+3)(3d-5)}{(4d-1)(3d-5)} = \frac{(2d+3)}{(4d-1)} \quad d \neq \frac{1}{4}, \frac{5}{3}$$

$$41. \quad \frac{40+6t-18t^2}{9t^2-9t-10} = \frac{-2(9t^2-3t-20)}{(3t-5)(3t+2)} = \frac{-2(3t-5)(3t+4)}{(3t-5)(3t+2)} = \frac{-2(3t+4)}{(3t+2)} \quad t \neq \frac{5}{3}, -\frac{2}{3}$$

$$42. \quad \frac{4m}{m^2+3m} - \frac{2}{m+3} = \frac{4m(m+3) - [2 \cdot (m^2+3m)]}{(m^2+3m)(m+3)} = \frac{4m^2+12m-2m^2-6m}{(m^2+3m)(m+3)} =$$

$$= \frac{2m^2+6m}{(m^2+3m)(m+3)} = \frac{2m(m+3)}{(m^2+3m)(m+3)} = \frac{2m(m+3)}{m(m+3)(m+3)} = \frac{2}{(m+3)} \quad m \neq 0, -3$$

$$43. \quad \frac{5}{2n+6} - \frac{2}{3n+9} = \frac{5(3n+9) - 2(2n+6)}{(2n+6)(3n+9)} = \frac{15n+45-4n-12}{(2n+6)(3n+9)} = \frac{11n+33}{2(n+3)3(n+3)} =$$

$$= \frac{11(n+3)}{6(n+3)(n+3)} = \frac{11}{6(n+3)} \quad n \neq -3$$

$$44. \quad \frac{2x+1}{x^2-x-2} + \frac{3}{x-2} = \frac{(2x+1)(x-2) + 3(x^2-x-2)}{(x^2-x-2)(x-2)} = \frac{2x^2-4x+x-2+3x^2-3x-6}{(x^2-x-2)(x-2)}$$

$$= \frac{5x^2-6x-8}{(x^2-x-2)(x-2)} = \frac{(5x+4)(x-2)}{(x^2-x-2)(x-2)} = \frac{(5x+4)}{(x^2-x-2)} = \frac{5x+4}{(x+1)(x-2)} \quad x \neq -1, 2$$

$$45. \quad \frac{4k}{k^2-9} - \frac{2}{k+3} = \frac{4k(k+3) - 2(k^2-9)}{(k+3)(k-3)(k+3)} = \frac{4k^2+12k-2k^2+18}{(k+3)(k-3)(k+3)} =$$

$$= \frac{2k^2+12k+18}{(k+3)(k-3)(k+3)} = \frac{2(k+3)^2}{(k+3)(k-3)(k+3)} = \frac{2}{k-3} \quad k \neq -3, 3$$

$$46. \quad \frac{1}{m+4} - \frac{m-3}{m^2+3m-4} = \frac{(m^2+3m-4) - [(m-3)(m+4)]}{(m+4)(m^2+3m-4)} =$$

$$= \frac{m^2+3m-4-m^2-4m+3m+12}{(m+4)(m^2+3m-4)} = \frac{2m+8}{(m+4)(m^2+3m-4)} =$$

$$= \frac{2(m+4)}{(m+4)(m+4)(m-1)} = \frac{2}{(m+4)(m-1)} \quad m \neq -4, 1$$

$$47. \quad \frac{3a+1}{2a-3} + \frac{a}{4a^2-9} = \frac{(3a+1)(4a^2-9) + a(2a-3)}{(2a-3)(4a^2-9)} =$$

$$= \frac{(3a+1)(2a-3)(2a+3) + a(2a-3)}{(2a-3)(2a-3)(2a+3)} = \frac{(2a-3)[(3a+1)(2a+3) + a]}{(2a-3)(2a-3)(2a+3)} =$$

$$= \frac{(3a+1)(2a+3) + a}{(2a-3)(2a+3)} = \frac{6a^2+3a+2a+3+a}{(2a-3)(2a+3)} = \frac{6a^2+12a+3}{(2a-3)(2a+3)} \quad a \neq -\frac{3}{2}, \frac{3}{2}$$

$$\begin{aligned}
 48. \quad \frac{3x}{x^2-1} - \frac{x}{x^2-2x+1} &= \frac{3x(x^2-2x+1) - x(x^2-1)}{(x^2-1)(x^2-2x+1)} = \frac{3x(x+1)(x-1) - x(x+1)(x-1)}{(x+1)(x-1)(x+1)(x+1)} = \\
 &= \frac{(x+1)[3x(x+1) - x(x-1)]}{(x+1)(x-1)(x+1)(x+1)} = \frac{3x(x+1) - x(x-1)}{(x-1)(x+1)(x+1)} = \frac{3x^2 + 3x - x^2 + x}{(x-1)(x+1)(x+1)} = \\
 &= \frac{2x^2 + 4x}{(x-1)(x+1)(x+1)} = \frac{2x(x+2)}{(x-1)(x+1)(x+1)} \quad x \neq -1, 1
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{3x}{x^2+x-2} + \frac{2}{x^2+3x-2} &= \frac{3x(x^2+3x-2) - 2(x^2+x-2)}{(x^2+x-2)(x^2+3x-2)} = \\
 &= \frac{3x(x+2)(x-1) + 2x(x+2)(x+1)}{(x+2)(x-1)(x+2)(x+1)} = \frac{3x(x+1) + 2x(x-1)}{(x-1)(x+2)(x+1)} = \frac{3x^2 + 3x + 2x^2 - 2x}{(x-1)(x+2)(x+1)} = \\
 &= \frac{5x^2 + x}{(x-1)(x+2)(x+1)} = \frac{x(5x+1)}{(x-1)(x+2)(x+1)} \quad x \neq -1, -2
 \end{aligned}$$

$$\begin{aligned}
 50. \quad \frac{5k}{k^2-3k-4} - \frac{3k}{k^2-2k-8} &= \frac{5k(k^2-2k-8) - 3k(k^2-3k-4)}{(k^2-3k-4)(k^2-2k-8)} = \\
 &= \frac{5k(k-4)(k+2) - [3k(k-4)(k+1)]}{(k-4)(k+1)(k-4)(k+2)} = \frac{(k-4)\{5k(k+2) - 3k(k+1)\}}{(k-4)(k+1)(k-4)(k+2)} = \\
 &= \frac{5k^2 + 10k - 3k^2 - 3k}{(k+1)(k-4)(k+2)} = \frac{k(2k+1)}{(k+1)(k-4)(k+2)} \quad k \neq -2, -1, 4
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \frac{2m+1}{m^2+2m-15} + \frac{1-3m}{m^2+4m-5} &= \frac{(2m-1)(m^2+4m-5) + (1-3m)(m^2+2m-15)}{(m^2+2m-15)(m^2+4m-5)} = \\
 &= \frac{(2m-1)(m+5)(m-1) + (1-3m)(m-3)(m+5)}{(m-3)(m+5)(m+5)(m-1)} = \\
 &= \frac{(m+5)[(2m-1)(m-1) + (1-3m)(m-3)]}{(m+5)(m-3)(m+5)(m-1)} = \frac{2m^2 - 2m + m - 1 + m - 3 - 3m^2 + 9m}{(m-3)(m+5)(m-1)} = \\
 &= \frac{2m+1}{m^2+2m-15} + \frac{1-3m}{m^2+4m-5} = \frac{-m^2 + 9m - 3}{(m-3)(m+5)(m-1)} \quad m \neq -5, 1, 3
 \end{aligned}$$

$$52. \quad \frac{5x+10}{x^2-3x-4} + \frac{2x+1}{3x^2-12x} = \frac{(5x+10)(3x^2-12x) + (2x+1)(x^2-3x-4)}{(x^2-3x-4)(3x^2-12x)} =$$

$$\begin{aligned}
 &= \frac{(5x+10)3x(x-4) + (2x+1)(x-4)(x+1)}{(x-4)3x(x-4)(x+1)} = \frac{(x-4)[(5x+10)3x + (2x+1)(x+1)]}{(x-4)3x(x+1)(x-4)} = \\
 &= \frac{15x^2 - 30x + 2x^2 + 2x + x + 1}{3x(x-4)(x+1)} = \frac{17x^2 + 33x + 1}{3x(x-4)(x+1)} \quad x \neq 0, -1, 4
 \end{aligned}$$

$$\begin{aligned}
 53. \quad &\frac{4a+5}{a^2-2a-3} - \frac{4-a}{3a^2-9a} = \frac{(4a+5)(3a^2-9a) - (4-a)(a^2-2a-3)}{(a^2-2a-3)(3a^2-9a)} = \\
 &= \frac{(4a+5)3a(a-3) - (4-a)(a-3)(a+1)}{(a-3)(a+1)3a(a-3)} = \frac{(a-3)[(4a+5)3a - (4-a)(a+1)]}{(a-3)(a+1)3a(a-3)} = \\
 &= \frac{12a^2 + 15a - 4a - 4 + a^2 + a}{(a+1)3a(a-3)} = \frac{13a^2 + 12a - 4}{(a+1)3a(a-3)} \quad a \neq -1, 0, 3
 \end{aligned}$$

$$\begin{aligned}
 54. \quad &\frac{3k}{6k^2+13k-5} + \frac{2k+1}{6k^2+7k-3} = \frac{3k(6k^2+7k-3) + (2k+1)(6k^2+13k-5)}{(6k^2+13k-5)(6k^2+7k-3)} = \\
 &= \frac{3k(3k-1)(2k+3) + (2k+1)(3k-1)(2k+5)}{(3k-1)(2k+5)(3k-1)(2k+3)} = \\
 &= \frac{(3k-1)[3k(2k+3) + (2k+1)(2k+5)]}{(3k-1)(2k+5)(3k-1)(2k+3)} = \frac{6k^2 + 9k + 4k^2 + 10k + 2k + 5}{(2k+5)(3k-1)(2k+3)} = \\
 &= \frac{10k^2 + 21k + 5}{(2k+5)(3k-1)(2k+3)} \quad k \neq -\frac{2}{5}, -\frac{3}{2}, \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad &\frac{4n^2-25}{8n^2+14n-15} \times \frac{12n^2-n-6}{6n^2+4n} = \frac{(2n-5)(2n+5)}{(4n-3)(2n+5)} \times \frac{(4n-3)(3n+2)}{2n(3n+2)} = \\
 &= \frac{2n-5}{2n} \quad n \neq -\frac{5}{2}, 0
 \end{aligned}$$

$$56. \quad \frac{8t^2+24t}{3t^2+4t-15} \div \frac{12t^2+8t}{9t-15} = \frac{8t(t+3)}{(3t-5)(t+3)} \times \frac{3(3t-5)}{4t(3t+2)} = \frac{6}{3t+2} \quad t \neq -\frac{2}{3}$$

$$\begin{aligned}
 57. \quad &\frac{4-5a}{24a^2+2a-12} - \frac{5-4a}{12a^2-15a-18} = \\
 &= \frac{(4-5a)[(4a+3)(3a-6)] - (5-4a)[(6a-4)(4a+3)]}{(6a-4)(4a+3)(4a+3)(3a-6)} =
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(4a+3)\left[(12a-24-15a^2+30a)-(30a-20-24a^2+16a)\right]}{(4a+3)(6a-4)(4a+3)(3a-6)} = \\
 &= \frac{9a^2-4a-4}{(6a-4)(4a+3)(3a-6)} = \frac{9a^2-4a-4}{2(3a-2)(4a+3)3(a-2)} = \frac{9a^2-4a-4}{(3a-2)(4a+3)(a-2)} \\
 &a \neq -\frac{3}{2}, -\frac{4}{4}, 2
 \end{aligned}$$

$$\begin{aligned}
 58. \quad &\left(\frac{a-3}{a^2-4} + \frac{a-1}{2a+4}\right) \times \frac{2a}{3a-6} = \left[\frac{(a-3)(2a+4) + (a-1)(a^2-4)}{(a^2-4)(2a+4)}\right] \times \frac{2a}{3a-6} = \\
 &= \frac{(a-3)2(a+2) + (a-1)(a-2)(a+2)}{2(a+2)(a-2)(a+2)} \times \frac{2a}{3a-6} = \\
 &= \frac{(a+2)\left[(a-3)2 + (a-1)(a-2)\right]}{2(a+2)(a-2)(a+2)} \times \frac{2a}{3(a-2)} = \frac{a(2a-6+a^2-2a-a+2)}{3(a+2)(a-2)(a-2)} = \\
 &= \frac{a(a^2-a-4)}{3(a+2)(a-2)(a-2)} \quad a \neq -2, 2
 \end{aligned}$$

$$\begin{aligned}
 59. \quad &(y+2)(3y+1) - (2y-1)(y+5) = 3y^2 + y + 6y + 2 - (2y^2 + 10y - y - 5) = \\
 &= 2y^2 - 2y + 7
 \end{aligned}$$

$$60. \quad \frac{y^2+4y-21}{y^2-6y-16} \times \frac{y^2-8y+15}{y^2+9y+14} = \frac{(y-3)(y+7)}{(y-8)(y+2)} \times \frac{(y-3)(y-5)}{(y+7)(y+2)} = \frac{(y-3)^2(y-2)}{(y+2)^2(y-8)}$$

$$\begin{aligned}
 61. \quad &\frac{n}{n-1} - \frac{2n}{n^2-1} = \frac{n(n^2-1) - 2n(n-1)}{(n-1)(n-1)(n+1)} = \frac{(n-1)[n(n+1) - 2n]}{(n-1)(n-1)(n+1)} = \frac{n^2+1-2n}{(n-1)(n+1)} = \\
 &= \frac{n(n-1)}{(n-1)(n+1)} = \frac{n}{n+1} \quad n \neq -1, 1
 \end{aligned}$$

$$\begin{aligned}
 62. \quad &\frac{2k}{k^2-1} + \frac{k^2-3k}{k^2+4k+3} \times \frac{k^2+2k-3}{k-3} = \frac{2k}{k^2-1} + \frac{k(k-3)(k-1)(k+3)}{(k-3)(k+1)(k+3)} = \\
 &= \frac{2k}{k^2-1} + \frac{k(k-1)}{k+1} = \frac{2k(k+1) + k(k-1)(k-1)(k+1)}{(k+1)(k^2-1)} = \frac{2k + (k-1)(k-1)}{(k-1)(k+1)} = \\
 &= \frac{k(k^2-2k+3)}{(k-1)(k+1)} \quad k \neq -3, -1, 1, 3
 \end{aligned}$$