2. ARITMETICKI I GEOMETRIJSKI NIZ, RED, BINOMNI POUCAK

2.1 Aritmeticki niz

Opci oblik aritmetickog niza: $a_n = a_1 + (n-1)d$

Gdje je: $a_1 \Rightarrow$ prvi clan aritmetickog niza

$a_n \Rightarrow n-ti$ clan aritmetickog niza

$n \Rightarrow$ broj clanova niza

$d \Rightarrow$ razlika ili diferencija aritmetickog niza

Zbroj, suma, clanova aritmetickog niza racuna se: $S_n = \frac{n}{2}(a_1 + a_n)$

Gdje je: $S_n \Rightarrow$ Suma n clanova aritmtickog niza

$n, a_1, a_n \Rightarrow$ Vidi gornju definiciju

1. Nadji n-ti clan aritmetickog niza: 1,4,7,... $n = 8$

   $a_1 = 1,\ n = 8,\ d = a_2 - a_1 = 4 - 1 = 3 \Rightarrow a_8 = a_1 + (8-1)3 = 1 + 21 = 22$

2. Nadji n-ti clan aritmetickog niza: -6,-4,-2,... $n = 10$

   $a_1 = -6, n = 10 \rightarrow d = a_2 - a_1 = -4 - (-6) = 2 \Rightarrow a_{10} = a_1 + (10-1)2 = -6 + 18 = 12$

3. Nadji n-ti clan aritmetickog niza: 18,13,8,... $n = 17$

   $a_1 = 18, n = 10 \rightarrow d = a_2 - a_1 = 13 - 18 = -5 \Rightarrow a_{17} = 18 + (17-1)(-5) = -62$

4. Nadji 12-ti clan niza kome je prvi clan $a_1 = 18$ a razlika $d = 4$

   $a_{12} = 18 + (12-1)4 = 37 \qquad a_{12} = 37$

5. Nadji 25-ti clan niza kome je prvi clan $a_1 = b,\ d = 2b$:

   $a_{25} = b + (25-1)2b \Rightarrow a_{25} = b + 48b = 49b$

6. Nadji sumu clanova aritmetickog niza: $a_1 = 4,\quad a_{20} = 40,\quad n = 20,$

   $$S_{20} = \frac{n}{2}(a_1 + a_{20}) = \frac{10}{2}(4 + 40) = 440$$

7. Nadji sumu clanova aritmetickog niza: $a_1 = -2,\quad d = -\frac{1}{2},\quad n = 10,$

   $$a_{10} = -2 + (10-1)\left(-\frac{1}{2}\right) = -\frac{13}{2} \qquad S_{10} = \frac{10}{2}\left(-2 - \frac{13}{2}\right) = -\frac{85}{2}$$

8. Nadji broj clanova i sumu aritmetickog niza: $a_1 = 5, \quad a_n = 45, \quad d = 8,$

$$45 = 5 + (n-1)8 \Rightarrow \underline{n = 6} \qquad S_6 = \frac{6}{2}(5+45) = 3 \cdot 50 = 150 \Rightarrow \underline{S_6 = 150}$$

9. Nadji 20-clan aritmetickog niza: $a_1 = \frac{5}{3}, \quad n = 20, \quad S_{20} = \frac{40}{3},$

$$\frac{40}{3} = \frac{20}{2}\left(\frac{5}{3} + a_{20}\right) \Rightarrow \underline{a_{20} = -\frac{1}{3}} \qquad a_{20} = \frac{5}{3} + (20-1)d$$

10. Nadji nepoznate clanove aritmetickog niza: $n = 30, \quad d = 3, \quad S_{30} = 1875$

$$S_{30} = \frac{30}{2}(a_1 + a_{30}) \Rightarrow 15(a_1 + a_{30}) = 1875 \qquad (a_1 + a_{30}) = 125$$

$$a_{30} = a_1 + (30-1)3 = a_1 + 87 \qquad \underline{(a_1 - a_{30}) = -87}$$

$$2a_1 = 38 \Rightarrow \underline{a_1 = 19}$$

11. Nadji nepoznate clanove aritmetickog niza: $a_1 = 74, \quad d = -5, \quad a_n = -231$

$$-231 = 74 + (n-1)(-5) \Rightarrow (n-1) = 61 + 87 \Rightarrow \underline{n = 62}$$

$$S_{62} = \frac{n}{2}(a_1 + a_{62}) = \frac{62}{2}(74 - 231) = 31(-157) = -4867 \Rightarrow \underline{S_{62} = -4867}$$

12. Nadji nepoznate clanove aritmetickog niza: $a_1 = -5k, \quad d = \frac{k}{2}, \quad S_n = \frac{23}{2}k$

$$S_n = \frac{23k}{2} = \frac{n}{2}(a_1 + a_n) \Rightarrow \frac{23k}{2} = \frac{n}{2}(-5k + a_n)$$

$$a_n = a_1 + (n-1)d \Rightarrow \underline{a_n = -5k + (n-1)\frac{k}{2}}$$

$$\frac{23k}{2} = \frac{n}{2}\left[-5k + \left(-5k + (n-1)\frac{k}{2}\right)\right] \Rightarrow \frac{23k}{2} = -\frac{5kn}{2} - \frac{5kn}{2} + \frac{n^2k}{4} - \frac{nk}{4} \quad x/4$$

$$46k = -10kn - 10kn + n^2k - nk \Rightarrow n^2 - 21n - 46 = 0$$

$$n_{1,2} = \frac{21 \pm \sqrt{21^2 - 4 \cdot 1 \cdot 46}}{2} = \frac{21 \pm 25}{2} \quad n_1 = 23 \quad n_2 = -2 \text{ nema smisla}$$

$$a_{23} = -5k + (23-1)\frac{k}{2} = -5k + 11k = 6k$$

$$\underline{n = 23} \qquad \underline{a_{23} = 6k}$$

13. Nadji nepoznate clanove aritmetickog niza: $a_1 = -c, \quad S_n = 2b - 4c, \quad a_n = \frac{b}{2}$

1) $a_n = a_1 + (n-1)d \Rightarrow \frac{b}{2} = -c + (n-1)d$

2) $S_n = \frac{n}{2}(a_1 + a_n) \Rightarrow (2b - 4c) = \frac{n}{2}\left(-c + \frac{b}{2}\right)$

$2b - 4c = \frac{n}{4}(b - 2c) \Rightarrow \qquad 4(2b - 4c) = n(b - 2c)$

$n = \frac{4(2b - 4c)}{(b - 2c)} = 8$ $\quad \underline{n = 8}$ uvrsti u jednadzbu 1)

$\frac{b}{2} = -c + (n-1)d \Rightarrow \quad \frac{b}{2} = -c + (8-1)d \Rightarrow b = -2c + 7d$

$d = \frac{b + 2c}{14} = \frac{1}{14}(b + 2c) \qquad \underline{d = \frac{1}{14}(b + 2c)}$

14. Nadji nepoznate clanove aritmetickog niza: $a_6 = 56, \quad a_{10} = 72,$

$a_n = a_1 + (n-1)d \Rightarrow 56 = a_1 + (6-1)d \Rightarrow \ 72 = a_1 + (10-1)d \Rightarrow \underline{d = 4}$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{10}{2}(36 + 72) = 540 \ \underline{S_n = 540}$

15. Nadji nepoznate clanove aritmetickog niza: $a_4 = 2, \quad a_{10} = 0,$

$a_n = a_1 + (n-1)d \Rightarrow a_4 = a_1 + (4-1)d \Rightarrow 2 = a_1 + 3d$

$\underline{a_{10} = a_1 + (10-1)d} \qquad \underline{0 = a_1 + 9d} \Rightarrow a_1 = -9d = -9\left(-\frac{1}{3}\right) = 3$

$2 = -6d \Rightarrow \underline{d = -\frac{1}{3}}$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{10}{2}(3 + 0) = 15 \Rightarrow \underline{S_n = 15}$

16. Nadji sumu prvih 100 negativnih bojeva: $a_1 = -1, \quad a_{100} = -100$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{100}{2}(-1 - 100) = 5050 \qquad \underline{S_n = 5050}$

17. Nadji sumu prvih 200 produkata broja 5: $\qquad a_1 = 1 \cdot 5,$

$a_{200} = a_1 + (n-1)d = 5 + (200 - 1)5 = 5 + 995 = 1000$

$S_{200} = \frac{n}{2}(a_1 + a_{200}) = \frac{200}{2}(5 + 1000) = 100500$

2.2 Geometrijski niz

Opci oblik geometrijskog niza: $a_n = a_1 \cdot q^{n-1}$

Gdje je: $a_1 \Rightarrow$ prvi clan geometrijskog niza

$a_n \Rightarrow n-ti$ clan geometrijskog niza

$n \Rightarrow$ broj clanova niza

$q \Rightarrow$ kvocijent, omjer, geometrijskog niza

Zbroj, suma, clanova geometrijskog niza racuna se: $S_n = \dfrac{a_1\left(1-q^n\right)}{1-q} \quad (q \neq 1)$

Gdje je: $S_n \Rightarrow$ Suma n clanova geometrijskog niza

$n, a_1, q \Rightarrow$ Vidi gornju definiciju

Zavisno o velicini kvocijenta geometrijskog niza, niz moze biti divergirajuci i konvergirajuci:

Za $|q| < 1$ Geometrijski niz konvergira prema nuli

Za $|q| > 1$ Geometrijski niz divergira u beskonacnost

Suma beskonacnog broja clanova konvergirajuceg geometrijskog niza iznosi:

$$S_\infty = \frac{a_1}{1-q} \quad \left(|q| < 1\right)$$

18. Izracunaj nepoznate clanove geometrijskog niza: $a_1 = 8,\ a_2 = 4,\ a_3 = 2,$

$$q = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2} \qquad a_8 = a_1 q^{n-1} = 8\left(\frac{1}{2}\right)^{8-1} = \frac{8}{2^7} = \frac{1}{16}$$

19. Izracunaj prvih 5 clanova geometrijskog niza: $a_1 = 45, \quad q = \dfrac{1}{3}$

$$a_n = a_1 q^{n-1} \Rightarrow a_2 = 45\left(\frac{1}{3}\right)^{2-1} = \frac{45}{3} = 15 \qquad a_3 = 45\left(\frac{1}{3}\right)^{3-1} = \frac{45}{3^2} = 5$$

$$a_4 = 45\left(\frac{1}{3}\right)^{4-1} = \frac{45}{3^3} = \frac{5}{3} \qquad a_5 = 45\left(\frac{1}{3}\right)^{5-1} = \frac{45}{3^4} = \frac{5}{9}$$

20. Izracunaj prvih 5 clanova geometrijskog niza: $a_1 = 2, \quad q = 3$

$$a_n = a_1 q^{n-1} \Rightarrow a_2 = 2 \cdot 3^{2-1} = 2 \cdot 3 = 6 \qquad a_3 = 2 \cdot 3^{3-1} = 2 \cdot 3^2 = 18$$
$$a_4 = 2 \cdot 3^{4-1} = 2 \cdot 3^3 = 54 \qquad a_5 = 2 \cdot 3^{5-1} = 2 \cdot 3^4 = 162$$

21. Izracunaj sesti (6) clan geometrijskog niza: $n = 6, \frac{1}{2}, 1, 2, ...$

$$a_1 = \frac{1}{2}, \quad q = \frac{a_2}{a_1} = \frac{a_3}{a_2} = 2 \Rightarrow a_n = a_1 q^{n-1} \Rightarrow a_6 = \frac{1}{2} 2^{6-1} = \frac{1}{2} 2^5 = 16$$

22. Izracunaj sedmi (7) clan geometrijskog niza: $n = 7, \quad 125, -25, 5$

$$q = \frac{a_2}{a_1} = \frac{-25}{125} = -\frac{1}{5} \Rightarrow a_n = a_1 q^{n-1} \Rightarrow a_7 = 125\left(-\frac{1}{5}\right)^{7-1} = 125 \cdot \left(-\frac{1}{5}\right)^6 = \frac{1}{125}$$

23. Izracunaj sesti (6) clan geometrijskog niza: $n = 6, \quad a_1 = -27, \quad q = -\frac{1}{3}$

$$a_n = a_1 q^{n-1} \Rightarrow a_6 = (-27)\left(-\frac{1}{3}\right)^{6-1} = \frac{-27}{-3^5} = \frac{1}{9} \qquad \Rightarrow \qquad \underline{a_6 = \frac{1}{9}}$$

24. Izracunaj sumu prvih n clanova geometrijskog niza ako je zadano: $a_1 = \frac{1}{8}, q = 4, \quad n = 5$

$$S_n = \frac{a_1(1-q^n)}{1-q} = \frac{\left(\frac{1}{8}\right)(1-4^5)}{1-4} = \frac{1-1024}{8 \cdot (-3)} = \frac{-1023}{-24} = \frac{341}{8} \Rightarrow \qquad \underline{S_5 = \frac{341}{8}}$$

25. Izracunaj sumu prvih n clanova geometrijskog niza ako je: $a_1 = 192, \quad a_n = 6, \quad n = 6$

$$a_6 = a_1 q^{n-1} \Rightarrow 6 = 192 q^{6-1} = 192 q^5 \Rightarrow q^5 = \frac{6}{192} \qquad q = \sqrt[5]{\frac{6}{192}} = \sqrt[5]{\frac{1}{2^5}} = \frac{1}{2} \Rightarrow \underline{q = \frac{1}{2}}$$

$$S_n = \frac{a_1(1-q^n)}{1-q} = \frac{192\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = \frac{192\left(1-\frac{1}{64}\right)}{\frac{1}{2}} = \frac{192\frac{63}{64}}{\frac{1}{2}} = \frac{192 \cdot 63 \cdot 2}{64} = 378$$

$\underline{S_6 = 378}$

26. Izracunaj elemente geometrijskog niza koji nedostaju: $a_1, a_n, S_n, n, q : a_1 = \frac{1}{16}, q = 4, n = 6$

$$a_n = a_1 q^{n-1} \Rightarrow a_6 = a_1 q^{6-1} = \frac{1}{16} 4^5 = 64 \Rightarrow a_6 = 64$$

$$S_n=\frac{a_1\left(1-q^n\right)}{1-q}=\frac{\frac{1}{16}\left(1-4^6\right)}{1-4}=\frac{1365}{16}\Rightarrow S_6=\frac{1365}{16}$$

27. Izracunaj elemente geometrijskog niza koji nedostaju: $a_1, a_n, S_n, n, q : q=\frac{3}{2}, n=5, S_5=211$

$$S_5=211=\frac{a_1\left(1-q^5\right)}{1-q}=\frac{a_1\left[1-\left(\frac{3}{2}\right)^5\right]}{1-\frac{3}{2}}=a_1\frac{\left(\frac{2^5-3^5}{2^5}\right)}{-\frac{1}{2}}=a_1\frac{422}{32}\Rightarrow a_1=16$$

$$a_5=a_1q^{5-1}=16\left(\frac{3}{2}\right)^4=\frac{2^4\cdot3^4}{2^4}=3^4=81\Rightarrow \underline{a_5=81}$$

28. Izracunaj elemente geometrijskog niza koji nedostaju: a_1, a_n, S_n, n, q :

$$a_1=75, q=\frac{1}{5}, a_n=\frac{3}{25}$$

$$a_n=a_1q^{n-1}\Rightarrow\frac{3}{25}=75\left(\frac{1}{5}\right)^{n-1}\Rightarrow\left(\frac{1}{5}\right)^{n-1}=\frac{3}{1875}=\frac{1}{625}\Rightarrow 5^{n-1}=5^4\Rightarrow \underline{n=5}$$

$$S_4=\frac{a_1\left[1-\left(\frac{1}{5}\right)^5\right]}{1-\frac{1}{5}}=\frac{75\left(1-\frac{1}{3125}\right)}{\frac{4}{5}}=\frac{75\cdot3124\cdot781}{3125\cdot4}=\frac{75\cdot5\cdot781}{3125}\Rightarrow \underline{S_4=\frac{2343}{25}}$$

29. Izracunaj a_{10} ako je a_4=8 i a_7=16

$$a_4=a_1q^3=8\Rightarrow a_1=\frac{a_4}{q^3}=\frac{8}{q^3}\qquad a_7=a_1q^6=16\Rightarrow \underline{a_1=\frac{a_7}{q^6}=\frac{16}{q^6}}$$

$$\frac{8}{q^3}=\frac{16}{q^6}\Rightarrow\frac{q^6}{q^3}=\frac{16}{8}\Rightarrow \underline{q=2^{\frac{1}{3}}}$$

$$a_1=\frac{8}{q^3}=\frac{8}{\left(2^{\frac{1}{3}}\right)^3}=\frac{8}{2}=4\Rightarrow \underline{a_1=4}\qquad a_{10}=a_1q^9=4\left(2^{\frac{1}{3}}\right)^9=4\cdot2^{\frac{9}{2}}=31\Rightarrow \underline{a_{10}=32}$$

30. Izracunaj sumu beskonacnog geometrijskog reda ako za red oblika: $4+2+1+\frac{1}{2}+...$

$$q=\frac{a_2}{a_1}=\frac{2}{4}=\frac{1}{2}\quad \underline{q=\frac{1}{2}}\qquad S_\infty=\frac{a_1}{1-q}=\frac{4}{1-\frac{1}{2}}=\frac{4}{\frac{1}{2}}=8\quad \underline{S_\infty=8}$$

31. Izracunaj sumu beskonacnog geometrijskog reda ako je red oblika: $5+1+0.2+0.04+...$

$$q=\frac{a_2}{a_1}=\frac{1}{5}\Rightarrow \underline{q=\frac{1}{5}} \qquad S_\infty=\frac{a_1}{1-q}=\frac{5}{1-\frac{1}{5}}=\frac{5}{\frac{4}{5}}=\frac{25}{5}\Rightarrow \underline{S_\infty=\frac{25}{5}}$$

32. Izracunaj sumu beskonacnog geometrijskog reda ako je red oblika: $1+\frac{7}{8}+\frac{49}{64}+...$

$$q=\frac{a_2}{a_1}=\frac{7}{8}\Rightarrow \underline{q=\frac{7}{8}} \qquad S_\infty=\frac{a_1}{1-q}=\frac{1}{1-\frac{7}{8}}=\frac{1}{\frac{1}{8}}=8\Rightarrow \underline{S_\infty=8}$$

33. Izracunaj sumu beskonacnog geometrijskog reda ako je red oblika: $1+10^{-4}+10^{-4}+...$

$$q=\frac{a_2}{a_1}=10^{-4}\Rightarrow \underline{q=10^{-4}} \qquad S_\infty=\frac{a_1}{1-q}=\frac{1}{1-10^{-4}}=\frac{1}{0.9999}\Rightarrow \underline{S_\infty=\frac{10000}{9999}}$$

34. Izracunaj sumu beskonacnog geometrijskog reda oblika: $\left(2+\sqrt{3}\right)+1+\left(2-\sqrt{3}\right)+...$

$$q=\frac{a_3}{a_2}=2-\sqrt{3}\Rightarrow \underline{q=2-\sqrt{3}}$$

$$S_\infty=\frac{a_1}{1-q}=\frac{2+\sqrt{3}}{1-\left(2-\sqrt{3}\right)}=\frac{2+\sqrt{3}}{1-2+\sqrt{3}}=\frac{2+\sqrt{3}}{\sqrt{3}-1}\,\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\left(2+\sqrt{3}\right)\left(\sqrt{3}-1\right)}{3-1}=\frac{1}{2}\left(5+3\sqrt{3}\right)$$

$$\underline{S_\infty=\frac{1}{2}\left(5+3\sqrt{3}\right)}$$

35. U kvadrat stranice $a=20\ cm$ ucrtamo kvadrat sa vrhovima u polovistima stranice.
U taj kvadrat opet, upisemo slijedeci kvadrat, sa vrhovima u polovistima stranica a_1.
Ako predpostavimo, da se moze tako ucrtati beskonacni broj kvadrata, izracunaj kolika je povsina svih kvadata zajeno.

Povrsina prvog kvadrata: $P_1=a_1^2$

Duzina stranice drugog kvadrata $a_2=\sqrt{\left(\frac{a_1}{2}\right)^2+\left(\frac{a_1}{2}\right)^2}=\sqrt{2\left(\frac{a_1}{2}\right)^2}=\frac{a_1}{\sqrt{2}}=a_1\frac{\sqrt{2}}{2}$

Povrsina drugog kvadrata: $P_2=a_2^2=\left(a_1\frac{\sqrt{2}}{2}\right)^2=a_1^2\frac{2}{4}=\frac{a_1^2}{2}=P_1\frac{1}{2}\Rightarrow q=\frac{P_2}{P_1}=\frac{1}{2}$

$$P_\infty=S_\infty=\frac{P_1}{1-q}=\frac{P_1}{1-\frac{1}{2}}=\frac{P_1}{\frac{1}{2}}=2P_1 \qquad \underline{P_\infty=2P_1}$$

36. Krug radijusa r, podijelimo u osam (8) jednakig djelova. Is prvog presjecista pravca i kruznice povicemo okomicu na prvi slijedeci pravac (promjer). Iz te tocke opet povucemo okomicu na slijedeci pravac i tako do osmog pravca. Ako predpostavimo da smo podijelili krug u beskonacno mnogo dijelova, izracunaj duzinu Arhimedove spirale koja nastaje na taj nacin.

Svaki kruzni isjecak ima centralni kut od $\alpha = 45^\circ$

Prvi segment Arhimedove spirale ima duzinu: $a_1 = r\sin\alpha \Rightarrow \sin 45^\circ = \frac{\sqrt{2}}{2}$

Prvi "novi radijus" ima duzinu: $b = r\cos\alpha \Rightarrow \cos 45^\circ = \frac{\sqrt{2}}{2}$

Drugi segment Arhimedove spirale ima duzinu $a_2 = b\sin\alpha = r\sin\alpha\cos\alpha$

$$q = \frac{a_2}{a_1} = \frac{r\sin\alpha\cos\alpha}{r\sin\alpha} = \cos\alpha = \frac{\sqrt{2}}{2}$$

$$L_\infty = S_\infty = \frac{a_1}{1-q} = \frac{r\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{r\frac{\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}} = \frac{r\sqrt{2}}{2-\sqrt{2}}\,\frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{r\left(2\sqrt{2}+2\right)}{4+2} = \frac{r}{3}\left(1+\sqrt{2}\right)$$

$$\underline{L_\infty = \frac{r}{3}\left(1+\sqrt{2}\right)}$$

2.3 Aritmeticki i Geometrijski red

1. Zadan je zbroj $S_n = 4n^2 + n$. Izracunaj prva 4 clana reda.

   Izrazi oblik za opci $n -$ ti clan reda.

   $S_n = 4n^2 + n \Rightarrow$ $\quad S_1 = 4 \cdot 1^2 + 1 = 5 \quad a_1 = 5 \quad a_1 = 5$

   $S_2 = 4 \cdot 2^2 + 2 = 18 \quad S_2 = S_1 + a_2 \Rightarrow 18 = 5 + a_2 \Rightarrow a_2 = 13$

   $S_3 = 4 \cdot 3^2 + 3 = 39 \quad S_3 = S_2 + a_3 \Rightarrow 39 = 18 + a_3 \Rightarrow a_3 = 21$

   $S_4 = 4 \cdot 4^2 + 4 = 68 \quad S_4 = S_3 + a_4 \Rightarrow 68 = 39 + a_4 \Rightarrow a_4 = 29$

   Prva cetiri clana aritmetickog reda jesu: $a_1 + a_2 + a_3 + a_4 \equiv 5 + 13 + 21 + 29$.

   Opci oblik izgleda ovako: $a_1 = 5, \quad d = 8 \Rightarrow a_n = a_1 + (n-1)d = 5 + 8n - 8 = 8n - 3$

   $\underline{a_n = 8n - 3}$ Do istog rjesenja bi dosli i na ovaj nacin:

   $$a_n = S_n - S_{n-1} = 4n^2 + n - \left[4(n-1)^2 + (n-1)\right]$$

   $$a_n = 4n^2 + n - \left[4n^2 - 8n + 1 + n - 1\right] = 4n^2 - 4n^2 - 3 + 8n \Rightarrow \underline{a_n = 8n - 3}$$

2. Zadan je red $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \frac{1}{7\times 9} + ... + \frac{1}{(2n-1)(2n+1)}$. Izracunaj prve 3 sume reda i moguci oblik 4. sume. Izrazi opci oblik za S_n.

   $$S_1 = \frac{1}{3}$$

   $$S_2 = S_1 + \frac{1}{15} = \frac{1}{3} + \frac{1}{15} = \frac{5+1}{15} = \frac{6}{15} = \frac{2}{5}$$

   $$S_3 = S_2 + \frac{1}{35} = \frac{2}{5} + \frac{1}{35} = \frac{14+1}{35} = \frac{15}{35} = \frac{3}{7}$$

   Sume su znaci: $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$.

   Cetvrta suma ima oblik: $\frac{4}{9}$ Opci oblik sume izgleda ovako: $S_n = \frac{n}{2n+1}$

3. Izracunaj sumu prvih 25 clanova aritmetickog reda: $2 + 9 + 16 + 25 + ... +$

   $a_n = a_1 + (n-1)d \Rightarrow a_1 = 2; \quad d = (9-2) = (16-9) = 7; \quad n = 25$

   $a_n = a_{25} = a_1 + (n-1)d = 2 + (25-1)7 = 2 + 24\times 7 = 170 \Rightarrow \quad a_{25} = 170$

   Oznacimo sumu od 25 elemenata S: $S = 2 \; +9 \; +16 \; +... + 170$

   Ispisimo u obrnutom redu: $\underline{S = 170 + 16 \; +9 \; +... + 2}$ $(+)$ brojimo

   $2S = 172 + 172 + 172 + ... + 172$

   Ukupno imamo 25 suma po 172 ili : $S = 25 \times 172 = 2150$

   Trazena suma prvih 25 clanova iznosi 2150.

Ako ponovimo cjeli postupak koristeci opce brojeve, dobiti cemo opci oblik za sumu n clanova aritmetickog reda:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad S_{25} = \frac{25}{2}\left[2\cdot 2 + (25-1)7\right] = \frac{25}{2}(4 + 24\cdot 7) = 2150$$

4. Izracunaj sumu prvih 9 clanova geometrijskog reda: $2 + 6 + 18 + 54 + ...$

$$a_n = a_1 q^{n-1} \Rightarrow a_1 = 2; \quad q = \frac{a_2}{a_1} = \frac{a_3}{a_2} = 3; \quad n = 9 \Rightarrow a_9 = 2\cdot 3^{9-1} = 2\cdot 3^8 = 13{,}122$$

Oznacimo sumu od 9 elemenata S : $S = 2 + 6 + 18 + ... + 13{,}122$

Izrazimo $S \times q$ u obrnutom redu: $3S = 6 + 18 + ... + 13{,}122 + 39{,}966 \; (-)$

$$2S = -2 + 39{,}966$$

$$2S = 39{,}964 \Rightarrow \underline{S = 19{,}682}$$

Trazena suma prvih 9 clanova geometrijskog niza iznosi 19,682.

Ako ponovimo cjeli postupak koristeci opce brojeve, dobiti cemo opci oblik za sumu n clanova geometrijskog reda:

$$S_n = a\frac{q^n - 1}{q - 1},\; q \neq 1 \qquad S_9 = 2\frac{3^9 - 1}{3 - 1} = 2\frac{19{,}683 - 1}{2} = 19{,}682$$

2.4 Binomni Poucak

Promotrimo binomni izraz i njihov razvoj u faktore:

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

Uvedimo novu kategoriju:

Faktorijel, koji se oznacava sa $n!$ $\quad n! = n(n-1)(n-2)(n-3)...(3)(2)(1)$

Binomna formula tada glasi:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + ... + b^n$$

Daljnjim razvojem, dobijamo binomni red:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

1. Izracunaj:

$3! = 3\cdot 2\cdot 1 = 6$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

$\frac{4!}{2!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = \frac{24}{2} = 12$

$1! = 1$

$0! = 1$ Po definiciji

2. Pojednostavi slijedece izraze koristeci binomni poucak:

$$(2x+3)^5 = (2x)^5 + 5(2x)^4 3 + \frac{5\cdot 4}{2!}(2x)^3 3^2 + \frac{5\cdot 4\cdot 3}{3!}(2x)^2 3^3 + \frac{5\cdot 4\cdot 3\cdot 2}{4!}(2x)3^4 +$$

$$\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{5!}3^5 = 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$$

3. $$\left(2a-b^2\right)^6=\left(2a\right)^6+6\left(2a\right)^5\left(-b^2\right)+\frac{6\cdot5}{2!}\left(2a\right)^4\left(-b^2\right)^2+\frac{6\cdot5\cdot4}{3!}\left(2a\right)^3\left(-b^2\right)^3+$$
$$+\frac{6\cdot5\cdot4\cdot3}{4!}\left(2a\right)^2\left(-b^2\right)^4+\frac{6\cdot5\cdot4\cdot3\cdot2}{5!}\left(2a\right)\left(-b^2\right)^5+\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{6!}\left(-b^2\right)^6=$$
$$\left(2a-b^2\right)^6=64a^6-192a^5b^2+240a^4b^4-160a^3b^6+60a^2b^8-12ab^{10}+b^{12}$$

4. Razvi u binomni red koristeci poznatu formulu (prva cetiri clana):

$$\left(1+x\right)^8=1+8x+\frac{8\cdot7}{2!}x^2+\frac{8\cdot7\cdot6}{3!}x^3+\frac{8\cdot7\cdot6\cdot5}{4!}x^4+...=1+8x+28x^2+56x^3+...$$

5. $$\left(1-x\right)^{-2}=1+\left(-2\right)\left(-x\right)+\frac{\left(-2\right)\cdot\left(-3\right)}{2!}\left(-x\right)^2+\frac{\left(-2\right)\cdot\left(-3\right)\cdot\left(-4\right)}{3!}\left(-x\right)^3+...$$
$$\left(1-x\right)^{-2}=1+2x+3x^2+4x^3+...$$

6. $$\sqrt{1+x}=\left(1+x\right)^{\frac{1}{2}}=1+\frac{1}{2}x+\frac{\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}-1\right)}{2!}x^2+\frac{\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3+...$$
$$\sqrt{1+x}=\left(1+x\right)^{\frac{1}{2}}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}+..$$

7. $$\frac{1}{\sqrt{9-9x}}=\frac{1}{3\sqrt{1-x}}=\frac{1}{3}\left(1-x\right)^{-\frac{1}{2}}=$$
$$=\frac{1}{3}\left[1+\left(-\frac{1}{2}\right)\left(-x\right)+\frac{\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}-1\right)}{2!}\left(-x\right)^2+\frac{\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-x\right)^3+...\right]=$$
$$=\frac{1}{3}\left(1+\frac{x}{2}+\frac{3x^2}{8}+\frac{15x^3}{16}+...\right)$$

8. Ako je clan $r+1$ u izrazu $\left(\text{a+b}\right)^n$ zadan u obliku $\frac{\text{n}\left(\text{n-1}\right)\left(n-2\right)...\left(n-r+1\right)}{r!}a^{n-r}b^n$,

izrazi slijedece clanove: $a)\ b^5$ u izrazu $\left(a+b\right)^8$ $b)\ y^6$ u izrazu $\left(x+y\right)^{10}$

$$b^5=\frac{\text{n}\left(\text{n-1}\right)\left(n-2\right)...\left(n-r+1\right)}{r!}a^{n-r}b^n=\frac{8\cdot7\cdot6\cdot5\cdot4}{5!}a^3b^5=56a^3b^5$$
$$y^6=\frac{\text{n}\left(\text{n-1}\right)\left(n-2\right)...\left(n-r+1\right)}{r!}a^{n-r}b^n=\frac{10\cdot9\cdot8\cdot7\cdot6\cdot5}{6!}x^4y^6=210x^4y^6$$

9. Nadji peti cla u zadanom izrazu: $(2x - 3b)^{12}$

$$(2x - 3b)^{12}_{peti} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4!}(2x)^{8}(-3b)^{4} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4!}2^{8}(-3)^{4}x^{8}b^{4} = 10,264,320x^{8}b^{4}$$

10. Nadji peti cla u zadanom izrazu: $(a-b)^{14}$

$$(a-b)^{14}_{peti} = \frac{14 \cdot 13 \cdot 12 \cdot 11}{4!}a^{10}(-b)^{4} = 1001a^{10}b^{4}$$

11. Zadani izraz $V = A(1-r)^{n}$ razvij u red za $n = 5$:

$$V = A(1-r)^{5} =$$

$$= A\left[1 + 5(-r) + \frac{5 \cdot 4}{2!}(-r)^{2} + \frac{5 \cdot 4 \cdot 3}{3!}(-r)^{3} + \frac{5 \cdot 4 \cdot 3 \cdot 2}{4!}(-r)^{4} + \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5!}(-r)^{5} + \right] =$$

$$= A\left(1 - 5r + 10r^{2} - 10r^{3} + 5r^{4} - r^{5}\right)$$

$$V = A(1-r)^{5} = A\left(1 - 5r + 10r^{2} - 10r^{3} + 5r^{4} - r^{5}\right)$$